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# Chapter 1:- Basic Circuit Concepts  Module 1

(1)

"An electric circuit (or) electric networks is an inter-connection of electrical elements linked together in a closed path so that an electric current may continuosly flow."

* "Charge is the quantity of electricity responsible for electric phenomena".

* The time rate of charge constitutes an electric current

$$\boxed{i(t) = \frac{dq(t)}{dt}} \quad (or) \quad \boxed{q(t) = \int_{-\infty}^{t} i(x)\,dx.} \qquad C/Sec$$

* "Current is the time rate of flow of electric charge past a given point".

* "Electrical Potential (voltage) at any point in a charged conductor is defined as the work done to bring a unit +ve charge from ∞ to that point.

The unit of voltage is volt (V). (or) (E)

$$\boxed{V = \frac{dw}{dq}} \qquad J/Coulomb$$

* Power is defined as the rate at which, the work is done (w).

$$\boxed{P = EI = \frac{E^2}{R} = I^2R}$$

$$(or) \quad \boxed{P = VI = \frac{V^2}{R} = I^2R}$$

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* Energy is the Capacity to do work (kWh (or) unit).

$$W = EIt = \frac{E^2 t}{R} = I^2 R t.$$

Elements of an Electric Circuit :-

Electric Circuit Consists of 2 types of elements.

1) Active elements (or) Sources.
2) Passive elements (or) Sinks.

1) Active elements :- "They are the elements of a ckt which posses energy of their own & Can impart it to other elements of the ckt." [Independent Sources].

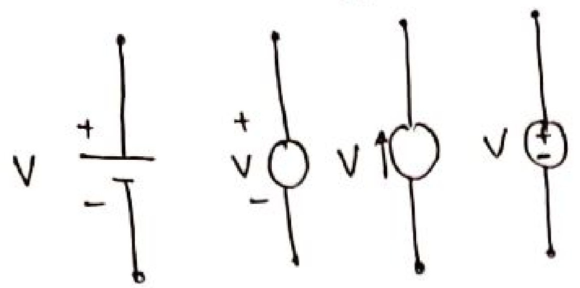
There are 2 types of active elements,

i) Voltage Source
ii) Current Source

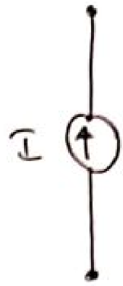
i) "An ideal Voltage Source is one, which delivers energy to a load at a Constant terminal voltage, irrespective of the Current drawn by the load."

ii) "An ideal Current Source is one, which delivers energy with a Constant Current to the load, irrespective of the terminal Voltage across the load".

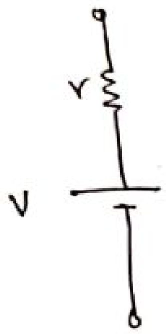
* Symbolically



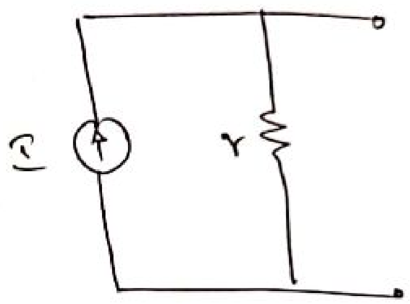

(a) Ideal D.C. Voltage Sources (Time Invariant Srcs)



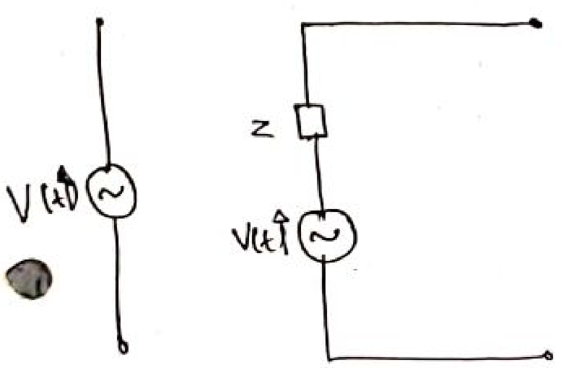

(b) Ideal D.C. Current Sources.

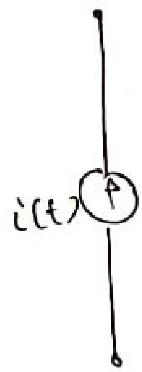



(c) Practical Voltage Source



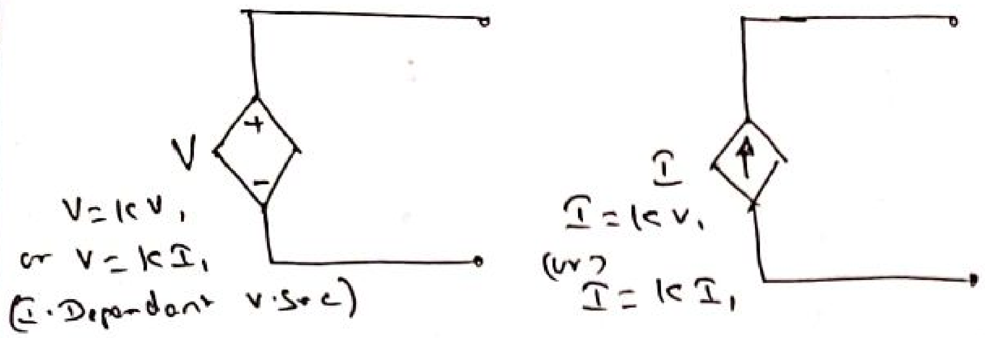

(d) Practical Current Source




(e) Ideal & Practical a.c. voltage Sources. (Time variant Srcs)

* Dependent Sources.




(a) Dependent Voltage Source (b) Dependent Current Source

Dependent Sources are Special kinds of Sources in which the Source voltage (or) Current depends upon a Current (or) voltage elsewhere in the ckt.

2) Passive Elements:- "These are the elements of an electric circuit which does not posses energy of their own." They receive energy from the Sources.

Eg:- Resistance, inductance & Capacitance.

* Resistance is the property of a Conductor by virtue of which it opposes (or) limits the flow of Current through it, unit is ohm ($\Omega$).

$$R = \frac{\rho l}{a}$$

$\rho \rightarrow$ Resistivity
$l \rightarrow$ length of Conductor.
$a \rightarrow$ Area of Cross section.

* A pure inductance does not Consume any power & the energy given to it is stored in the form of electromagnetic field & is given by

$$E = \frac{1}{2} L I^2 \text{ W sec}$$

$I \rightarrow$ Current flowing through inductor.

* A pure Capacitance does not Consumes any power & the energy given to it is stored in the form of electrostatic field & is given by

$$E = \frac{1}{2} C V^2 .$$

V → voltage applied across a Capacitor.

Ohm's Law:-

" The temperature remaining Constant, the Current flowing through any Conductor is directly proportional to the potential difference b/w the 2 ends of the Conductor".

$$I \propto V$$

$$I = \frac{V}{R}$$

$$\boxed{V = IR}.$$

Definitions :- P-14 N/ws & Systems D. Roy Chowdhury

1) Circuit Element:- Any individual ckt Component (Eg:- R, C, L) with 2 terminals, by which it Can be Connected to other electric Components.

2) Branch:- A group of ckt elements, usually in series & with 2 terminals

3) Potential src (Independent):- A hypothetical generator which maintains its value of potential independent of the o/p Current. An a.c. Source will be indicated by a $\bigcirc^{k}$ enclosing a wavy line

4) Current Src (Independant):- A generator which maintains its o/p Current independent of the voltage across its terminals. It is indicated by a Ckt enclosing an arrow for reference Current direction.

5) Network & Circuit:- An electric n/w is any possible interconnection of electric ckt elements (or) branches. An electric ckt is a closed energized n/w. A n/w is not necessarily a ckt. Eg:- T- n/w.

6) Lumped N/W:- A n/w in which physically seperate resistors, Capacitors & inductors can be represented.

7) Distributed N/W:- one in which resistors, Capacitors & inductors Cannot be electrically seperated & individually isolated as seperate elements.

Eg:- Transmission Line.

8) Passive n/w:- A n/w Containing ckt elements without any energy Sources.

9) Active n/w:- A n/w Containing energy sources together with other ckt elements.

10) Linear element:- A ckt element is linear if the relation b/w Current and Voltage involves a Constant Co-efficient.

Eg:- $v = Ri$ , $v = L\frac{di}{dt}$ , $v = \frac{1}{C}\int i\,dt$.

A linear n/w is one in which principle of Superposition holds. A nonlinear n/w is one which is not linear.

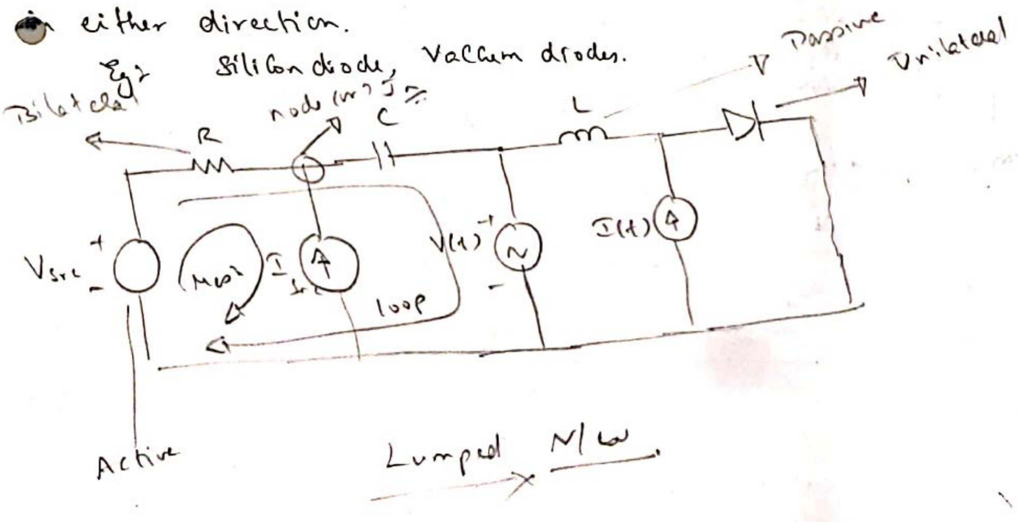
11) Mesh & Loop:- A set of branches forming a closed path, with the omission of any branch making the path open. Mesh must not have any other Clct inside it. Loop may have other loops (or) meshes inside it.

12) Node (or) junction:- A terminal of any branch of a n/w common to 2 (or) more branches is known as a node.
Voltage of any node w.r.t gnd is the node voltage.
Voltage b/w any pair of nodes is the node-pair voltage.

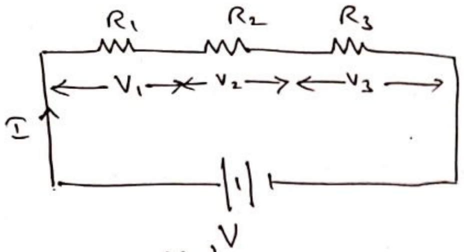
13) Vector & Phasor:- Vector is generalized multidimensional quantity having both magnitude & direction.
Phasor is a 2 dimensional vector used in electrical technology which relates to voltage & current.

14) Bilateral & unilateral elements:-
A bilateral element, the same relation exists b/w voltage & current flowing in either direction.
Eg:- voltage src, current source

An unilateral is one, in which the same relation does not (hold) exist b/w the voltage & current in either direction.
Eg:- Silicon diode, Vaccum diodes.



## Resistances in Series:-




(a)

Let $R_1$, $R_2$ & $R_3$ be 3 resistors Connected in Series. & $V_1$, $V_2$ & $V_3$ be their Corresponding voltage drops respectively

Let V be the total voltage applied & I be the Current flowing through the ckt.

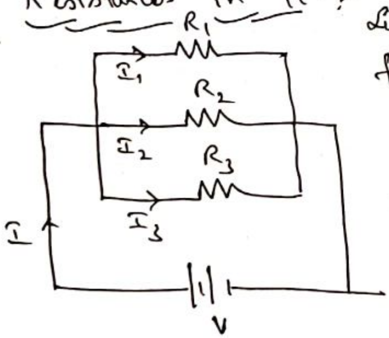
$$V = V_1 + V_2 + V_3$$

$$IR = IR_1 + IR_2 + IR_3$$

$$\boxed{R = R_1 + R_2 + R_3}$$ total resistance

$|||^{ly}$ with n resistor in series, the total resistance is $R = R_1 + R_2 + \cdots\cdots + R_n$.

## Resistances in ||$^{l}$:-




Let $I_1$, $I_2$ & $I_3$ be the currents flowing through $R_1$, $R_2$ & $R_3$ respectively

$$I = I_1 + I_2 + I_3$$

i.e. $$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\boxed{\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

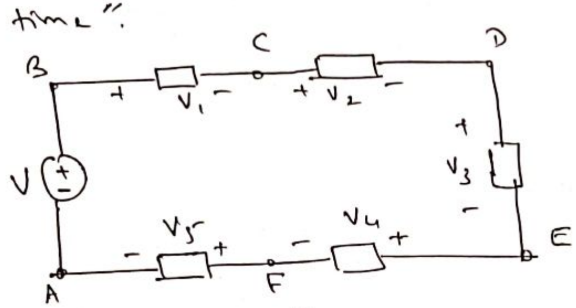
$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

Note: $R = \frac{R_1 R_2}{R_1 + R_2}$

## Kirchhoff's Laws:-

**(i) Kirchhoff's voltage Law:-**

"The algebraic sum of all branch voltages around any closed loop of a n/w is Zero at all instants of time".




(a)

Consider a electrical ckt with six nodes A, B, C, D, E & F as shown in figure

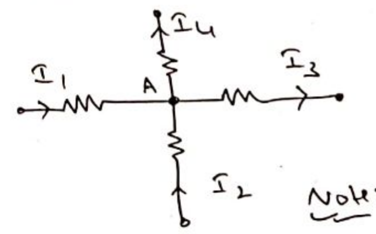
Applying KVL to the above ckt,

$$V - V_1 - V_2 - V_3 - V_4 - V_5 = 0$$

i.e $\Sigma V = 0$

**(ii) Kirchhoff's Current Law:-**

"The algebraic sum of branch Currents at a node is zero at all instants of time."




$$I_1 + I_2 - I_3 - I_4 = 0$$

$$\Sigma I = 0$$

Note:- Currents entering the node are taken as +ve & leaving the node as -ve.

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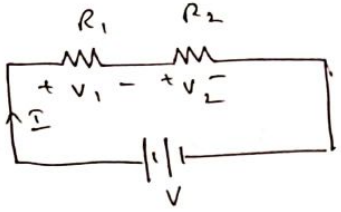



$V = V_1 + V_2$

$V = V_1 + I R_2$

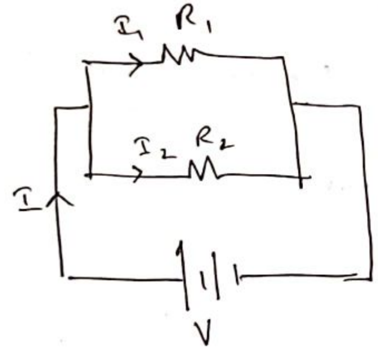
$V = V_1 + \frac{V}{R} R_2$

$V = V_1 + \frac{V}{R_1 + R_2} R_2$

$\therefore \boxed{V_1 = \frac{V R_1}{R_1 + R_2}}$

|||ly $\boxed{V_2 = \frac{V R_2}{R_1 + R_2}}$

*




$I = I_1 + I_2$

$I = I_1 + \frac{V}{R_2}$

$I = I_1 + \frac{I R}{R_2}$

$I_1 = I \left[ 1 - \frac{R}{R_2} \right]$

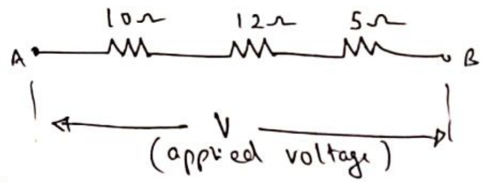
$I_1 = I \left[ 1 - \left[ \frac{R_1 R_2}{R_1 + R_2} \right] \Big/ R_2 \right]$

$I_1 = I \left[ \frac{R_1 + R_2 - R_1}{R_1 + R_2} \right]$

$\boxed{I_1 = \frac{I R_2}{R_1 + R_2}}$ |||ly $\boxed{I_2 = \frac{I R_1}{R_1 + R_2}}$

## Problems:

1)

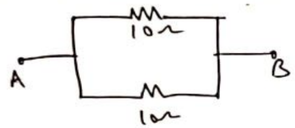



Sol:

$R_s = R_1 + R_2 + R_3 = 10 + 12 + 5$

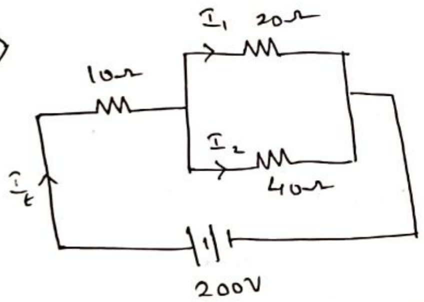
$\boxed{R_s = 27\Omega}$

2)




Sol: $R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 10}{10+10} = 5\Omega$

3)




Find the Current in each resistance & voltage across 10Ω. Find also the Power Consumed in all the resistances.

Sol: The net resistance is,

$$R_t = 10\Omega + \frac{20 \times 40}{20+40} = 23.33\Omega$$

$$\therefore I_t = \frac{V_t}{R_t} = \frac{200}{23.33} = 8.57A$$

$$I_1 = \frac{I \times 40}{20+40} = \frac{8.57 \times 40}{60} = 5.71A$$

IIIly $$I_2 = \frac{8.57 \times 20}{20+40} = 2.86A$$

$V_{10\Omega} = I \times 10 = 8.57 \times 10 = 85.7$ Volts //

$P = VI = I^2 R$

$P_{10\Omega} = (8.57)^2 \times 10 = 734.449W$ //

$P_{20\Omega} = (5.71)^2 \times 20 = 652.082 W$ //

$P_{40\Omega} = (2.86)^2 \times 40 = 327.184 W$ //

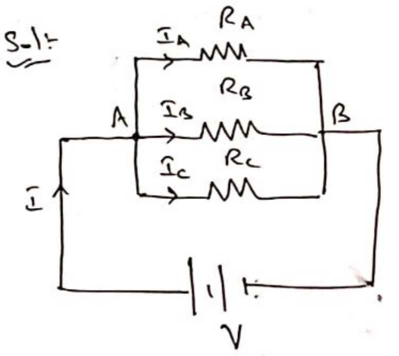
4) 3 resistors A, B & C are Connected in $\parallel^{l}$ taking a total Current of 12A from the supply. If $I_B = 2I_A$, $I_C = 3.5 I_B$ & the total ~~current~~ power drawn is 3KW, Calculate

(a) Current drawn by each resistor.

(b) Supply voltage

(c) Power Consumed by each resistor.

Sol:




(a) Apply KCL at node A

$I = I_A + I_B + I_C$

$12 = I_A + 2I_A + 3.5I_B$

$12 = I_A + 2I_A + 3.5(2I_A)$

$\therefore I_A = 1.2 A$ //

$I_B = 2I_A = 2(1.2) = 2.4A$ //

$I_C = 3.5 I_B = 8.4A$

(b) $P = VI$

$V = \frac{P}{I} = \frac{3k}{12} = 250V$ //

(c) $\cancel{P_A} = \cancel{I_A^2 R_A} = \cancel{k \times 2}$

$P = VI$

$P_A = VI_A = 300W$

$P_B = VI_B = 600W$

$P_C = VI_C = 2100W$

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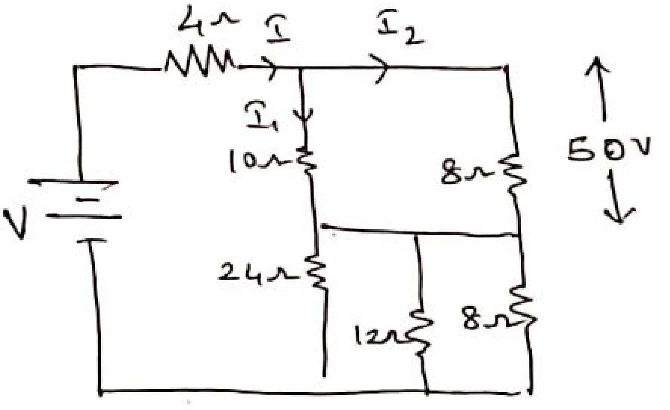
5) In the ckt given find the voltage drop across $4\,\Omega$ resistor & the supply voltage. (7)




Let $I$, $I_1$ & $I_2$ be the currents flowing in the branches shown in fig.

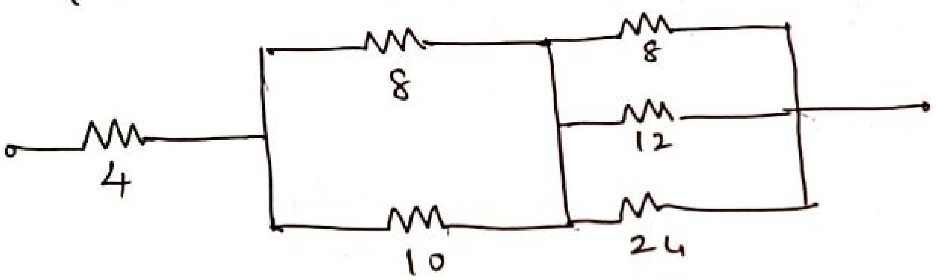
$$I_1 = \frac{50}{10} = 5A$$

$$I_2 = \frac{50}{8} = 6.25A$$

$$I = I_1 + I_2 = 11.25A$$

$$V_{4\Omega} = 4I = 4 \times 11.25 = 45V$$

$$R_t = (24 \parallel 12 \parallel 8) + (10 \parallel 8) + 4$$




$$24 \parallel 12 = \frac{24 \times 12}{24+12} = 8\Omega, \quad 8\Omega \parallel 8\Omega = \frac{8 \times 8}{8+8} = 4\Omega$$

$$10 \parallel 8 = \frac{10 \times 8}{10+8} = 4.44\Omega$$

$$R_t = 4 + 4.44 + 4 = 12.44\Omega$$

$$\therefore V = R_t I = 12.44 \times 11.25 = 140V$$

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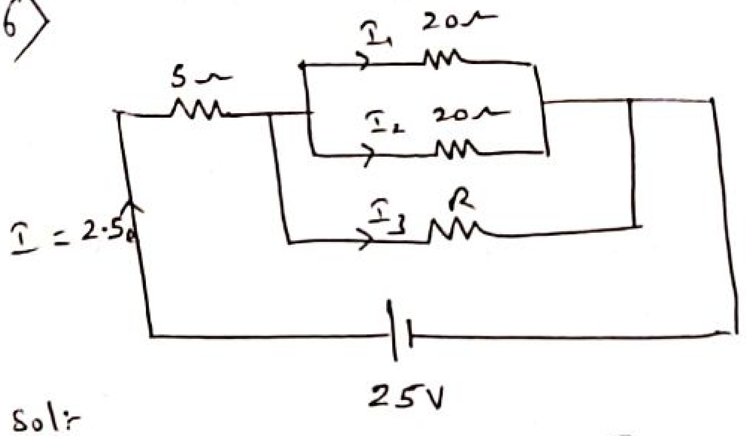
6) find the value of R.




Sol:-

$V_{5\Omega} = 2.5 \times 5 = 12.5$

Voltage across $\parallel^{l}$ Combination $= 25 - 12.5 = 12.5V.$

$I_1 = \frac{12.5}{20} = 0.625A = I_2$

But $I = I_1 + I_2 + I_3$

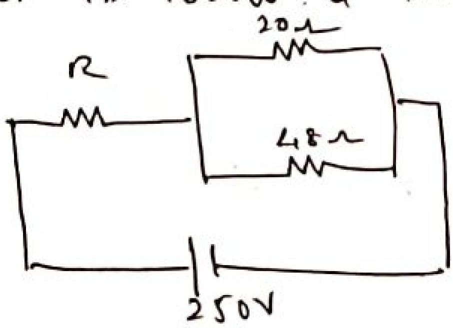
$2.5 = 0.625 + 0.625 + I_3$

$\therefore I_3 = 1.25A$

$\therefore R = \frac{12.5}{I_3} = \frac{12.5}{1.25} = 10\Omega$ $\boxed{R = 10\Omega}$

7) A resistance R is connected in series with a $\parallel^{l}$ ckt comprising 20Ω & 48Ω. The total power dissipated in the ckt is 1000W & the applied voltage is 250V. Calculate R.

Sol/.




$P = \frac{V^2}{R_t} = 1000$

$\frac{(250)^2}{R_t} = 1000$

$R_t = 62.5\Omega$

$R_t = R + \frac{20 \times 48}{20 + 48} = 62.5$

$\boxed{\therefore R = 48.38\Omega}$

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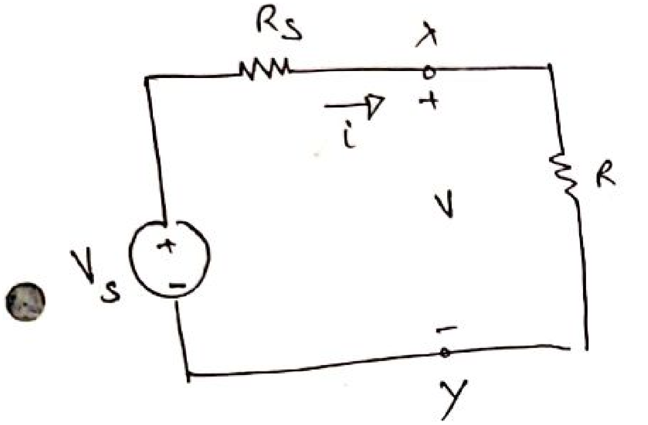
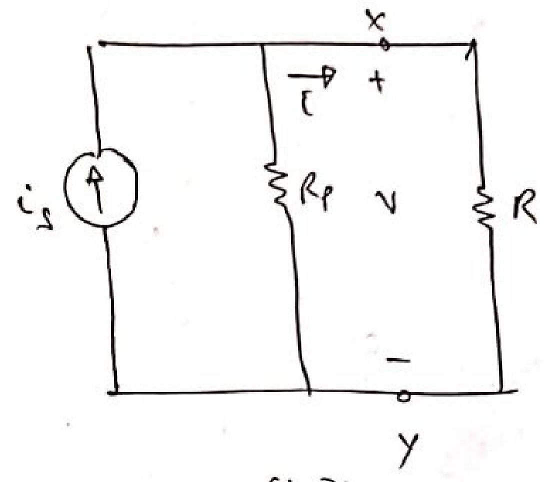
## Source Transformations:

* Src transformation is a procedure which transforms one Src into another while retaining the terminal characteristics of the original src
* An Equivalent ckt is one whose terminal characteristics remain identical to those of the original ckt.




(a)




(b)

* we want to transform the ckt in (a) to (b)

* for all values of R both the ckts should have same characteristics b/w the terminal x & y.

* If $R=0$, i.e xy short ckted.

In fig (a) $i = \frac{V_s}{R_s}$

In fig (b) the s.c current is $i_s$

$$\therefore \quad i_s = \frac{V_s}{R_s} \quad \text{———} (1)$$

* If $R = \infty$ i.e xy open ckted

In fig (b) $V = i_s R_p$

In fig (a) the o.c. voltage is $V_s$ $\therefore V_s = i_s R_p$ — (2

Sub eq (1) in eq (2)

$$V_s = \left(\frac{V_s}{R_s}\right) R_p$$

$$\boxed{\therefore R_s = R_p}$$

* Applying KVL to fig (a)

$$V_s - iR_s - V = 0$$

$$V_s = iR_s + V$$

$$\frac{V_s}{R_s} = i + \frac{V}{R_s} \quad \text{——— (3)}$$

Applying KCL to fig (b)

$$i_s = i + \frac{V}{R_p} \quad \text{——— (4)}$$

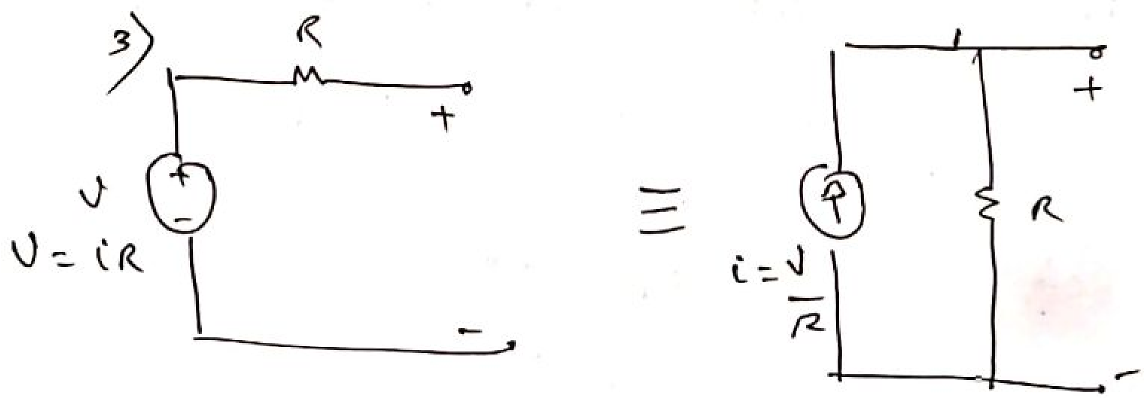
Thus fig (a) & (b) are equivalent if,

$$\boxed{i_s = \frac{V_s}{R_s} \quad \& \quad R_s = R_p}$$

Note: 1) A current src $i$ in $\|^{l}$ with a resistor R can be replaced by a voltage src of $V = iR$ in series with a resistor R.

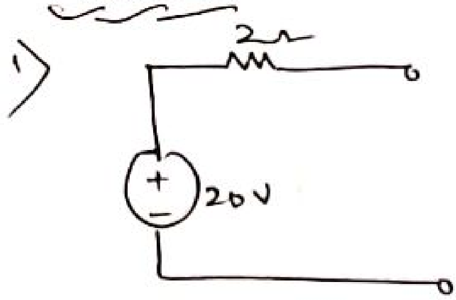
2) Reverse is also true.

3)



Problems:

1)

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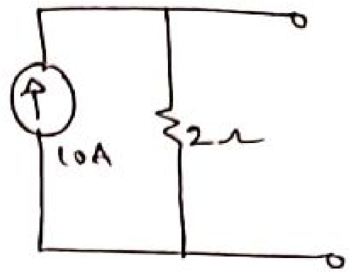
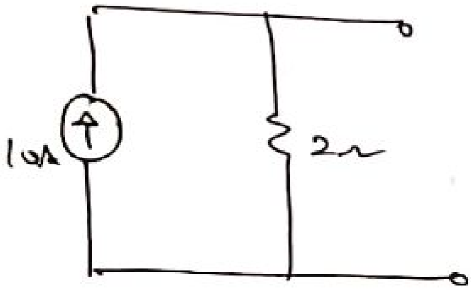
Sol




$$i = \frac{V}{R} = \frac{20}{2} = 10A$$

2)

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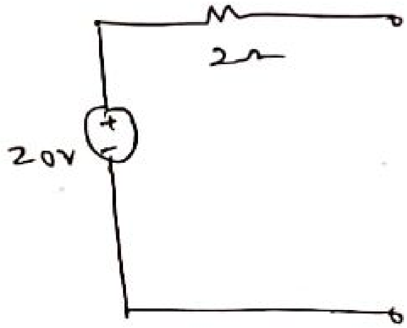
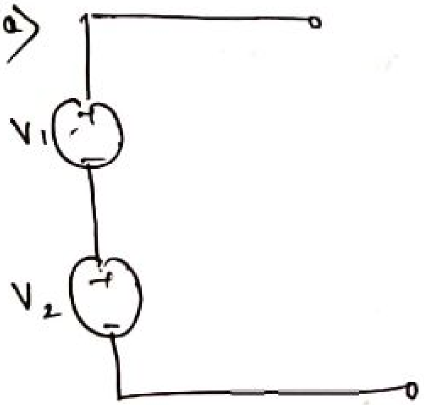



$$V = iR = 10 \times 2 = 20V$$

3) Note:

a)

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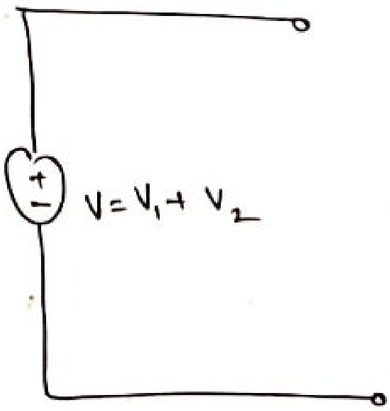
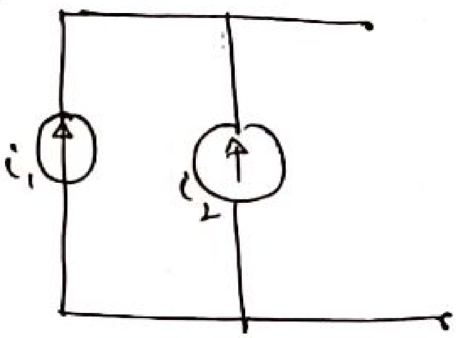



$V = V_1 + V_2$

b)

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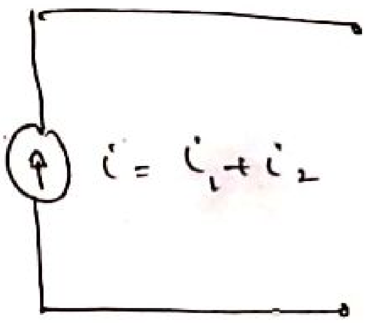
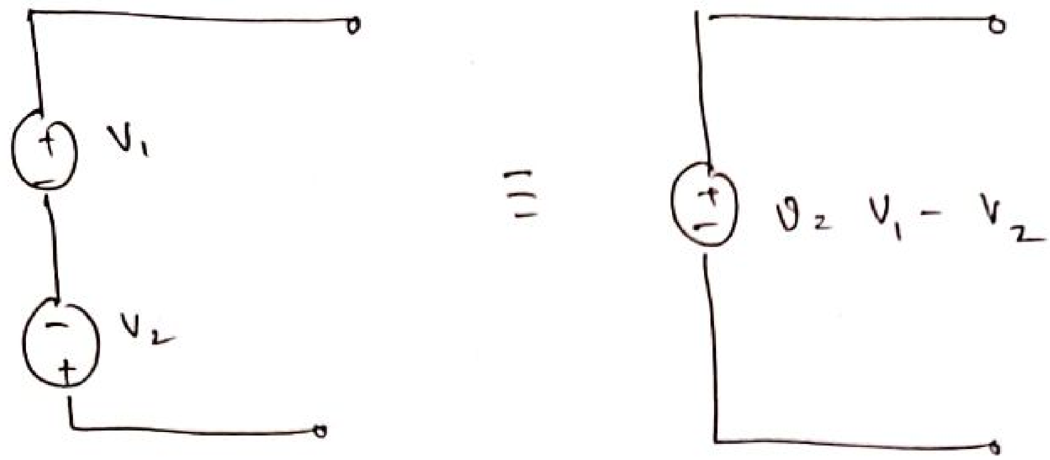


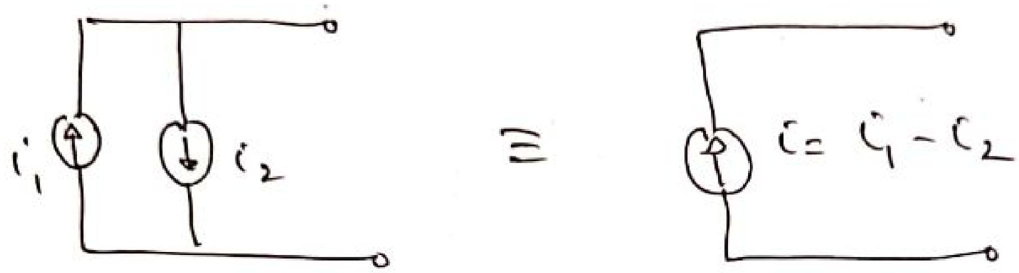
$i = i_1 + i_2$

c)



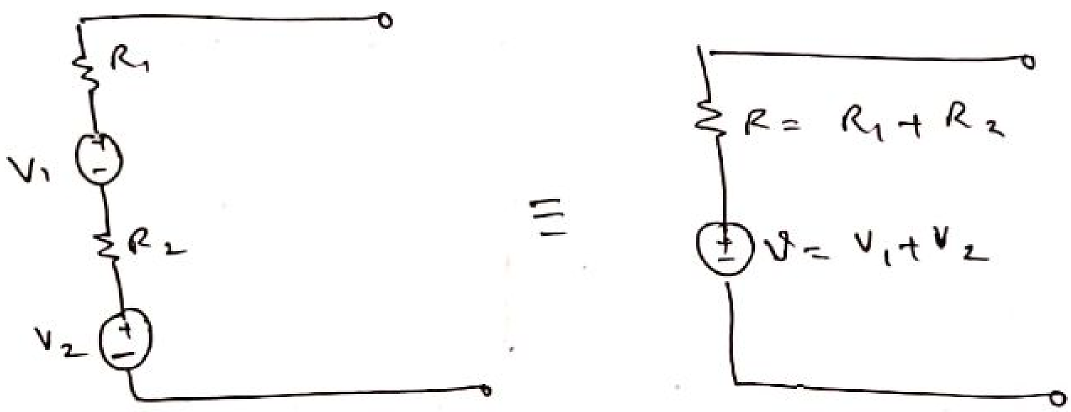
V1
V2
≡
v = V1 − V2


d)



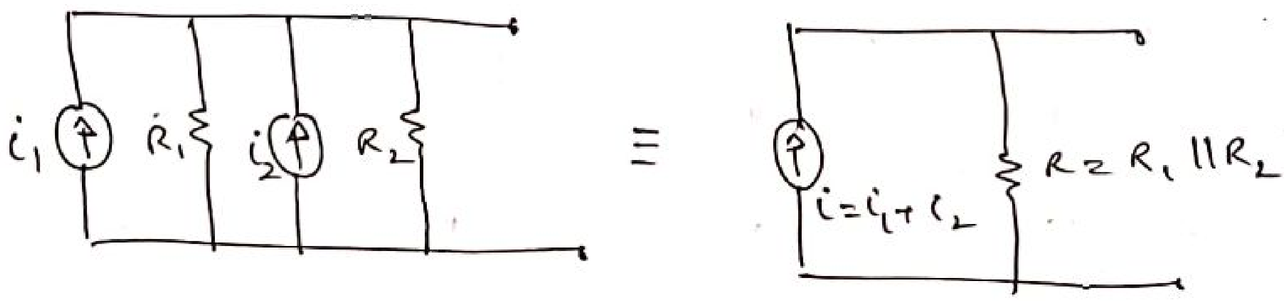
i1
i2
≡
i = i1 − i2


e)



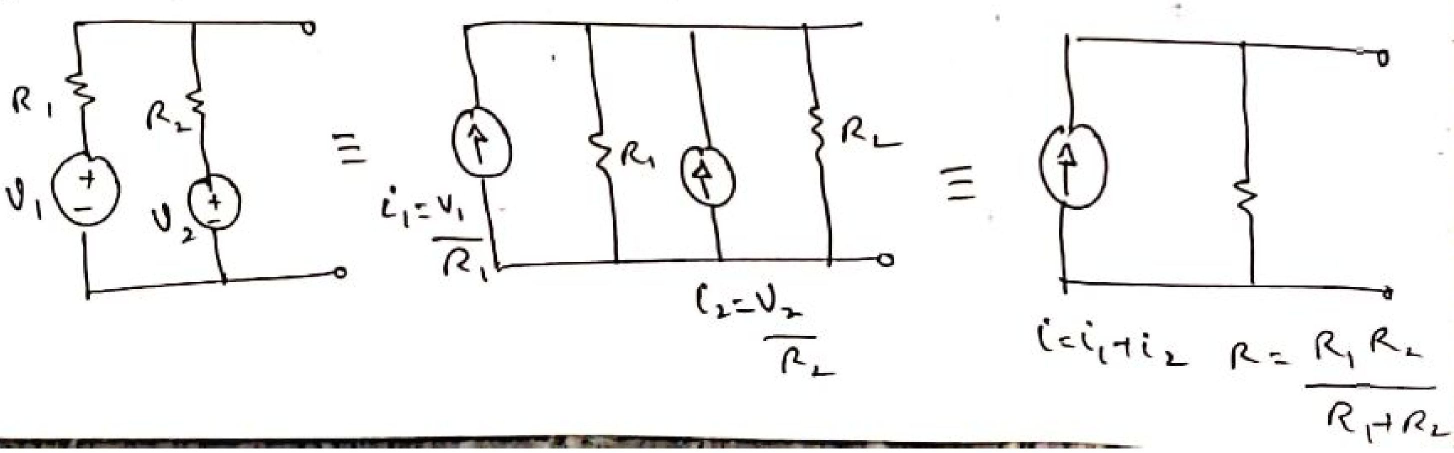
R1
V1
R2
V2
≡
R = R1 + R2
v = V1 + V2


f)



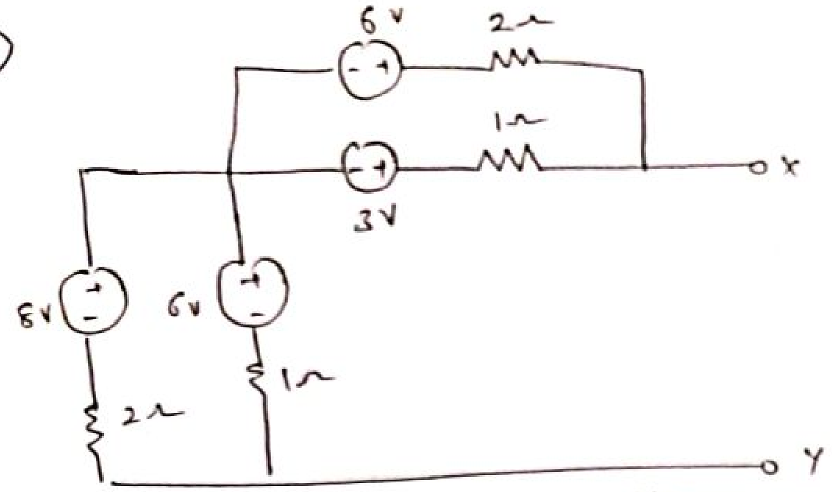
i1
R1
i2
R2
≡
i = i1 + i2
R = R1 || R2


g)



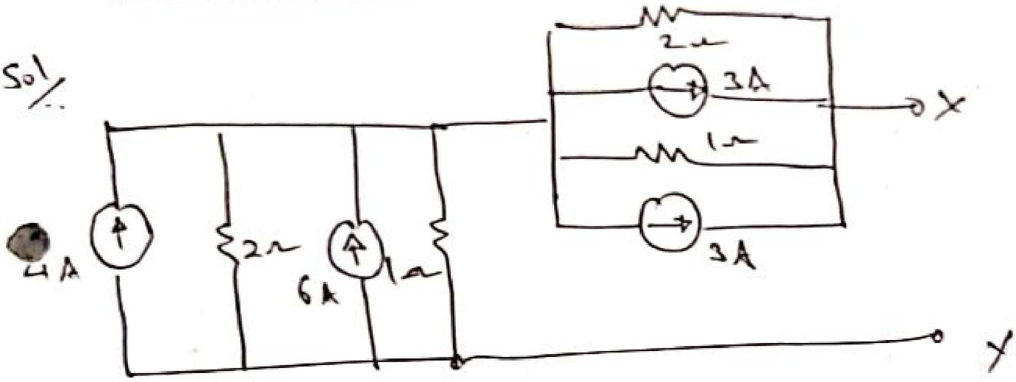
R1
R2
v1
v2
≡
i1 = v1/R1
R1
i2 = v2/R2
R2
≡
i = i1 + i2
R = R1R2/(R1+R2)

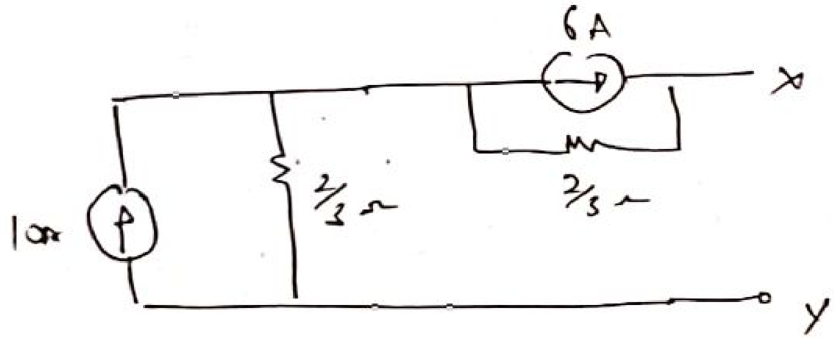
3)

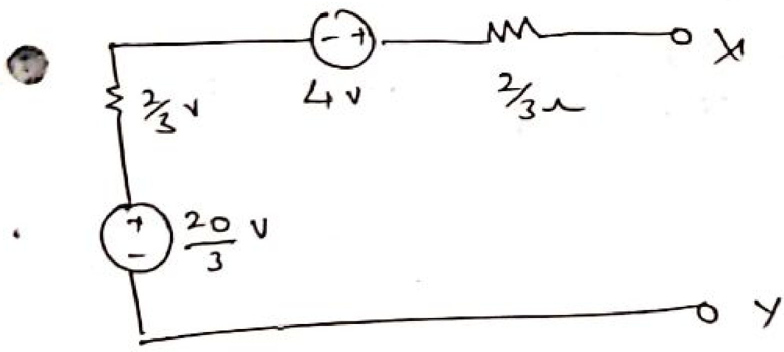


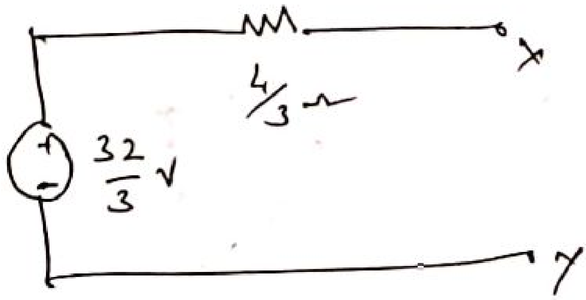

Reduce the ckt to a single Voltage src & a resistor.

Sol/.

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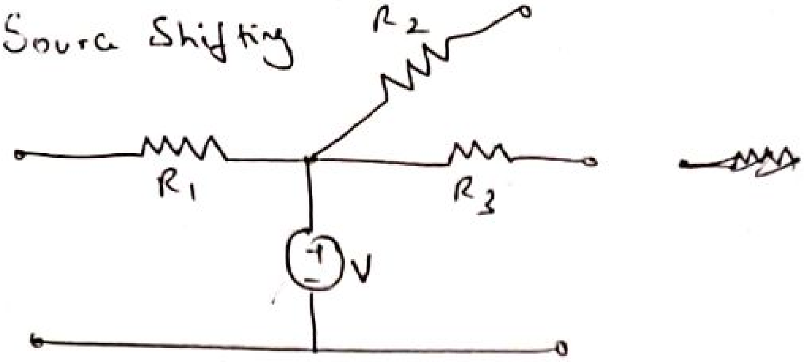
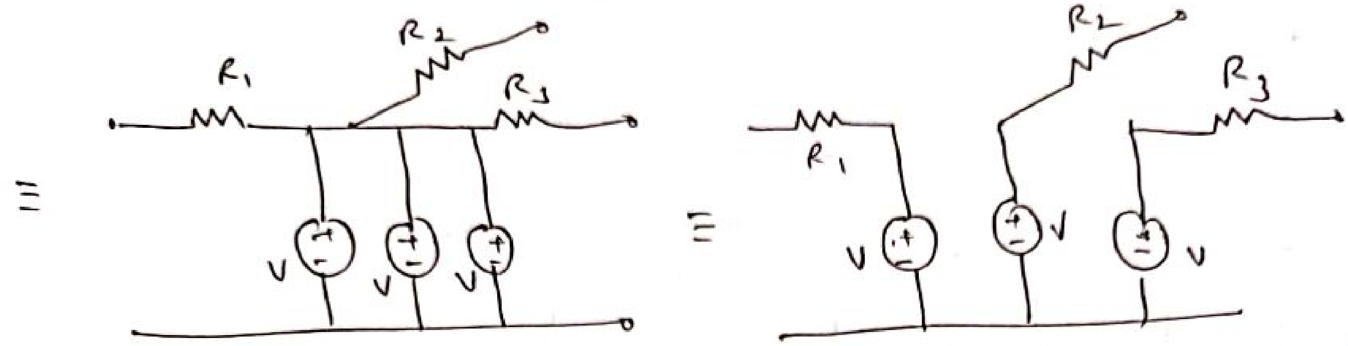
Note:

Source Shifting

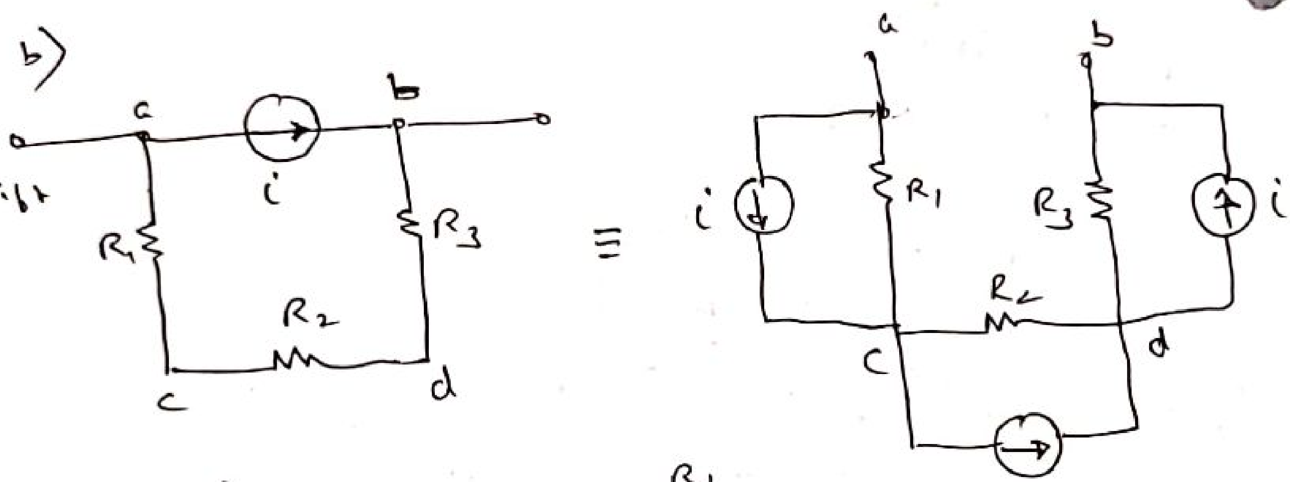
c) E-Shift



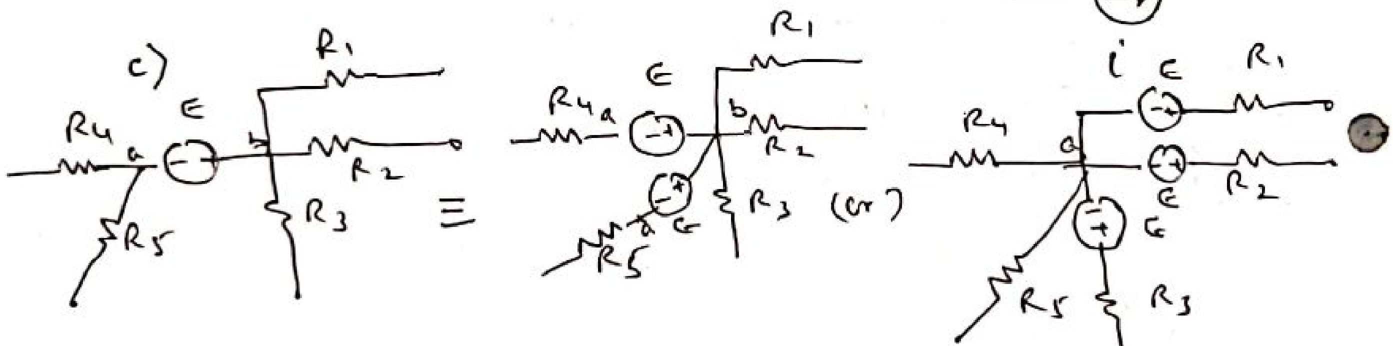




b)

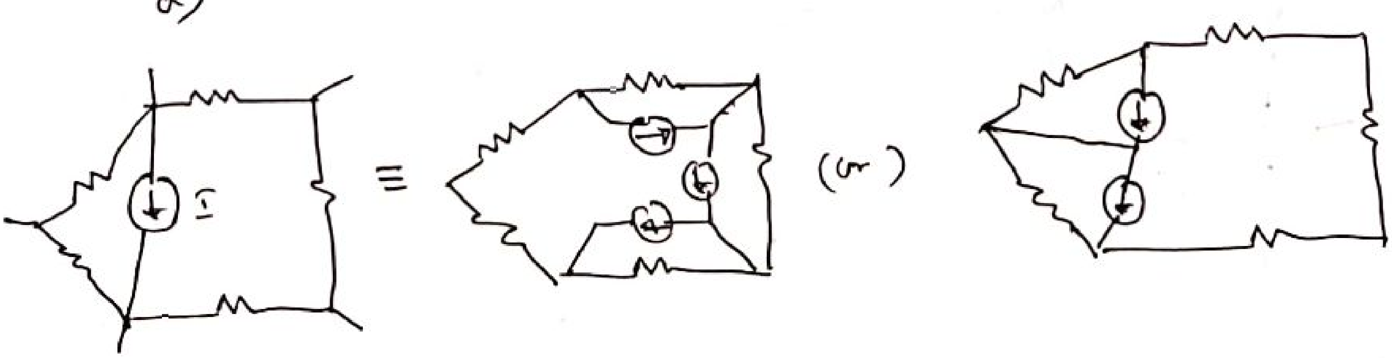
I - Shift



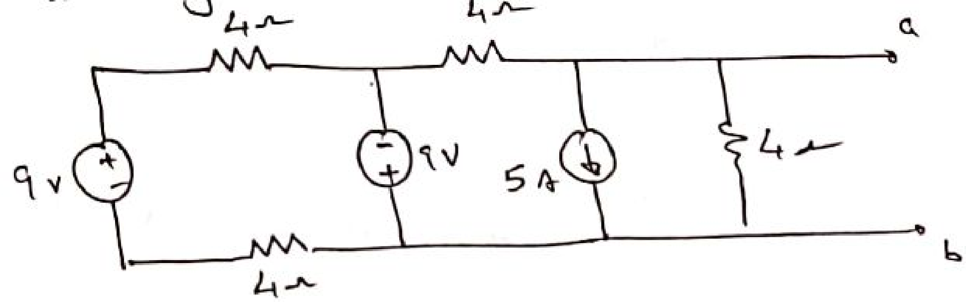

c)



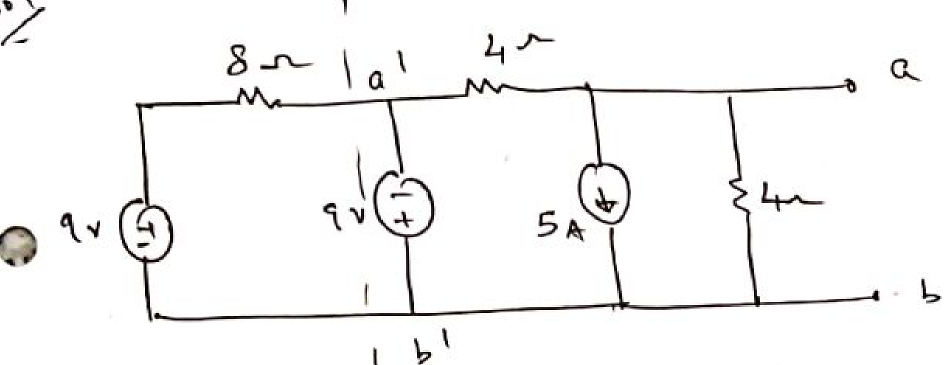

d)



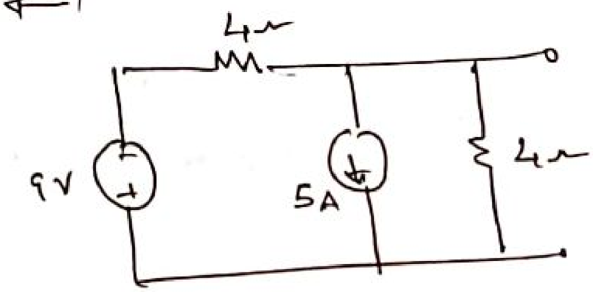
4) Reduce the ckt to a single current src & a single resistor in ||le to it.

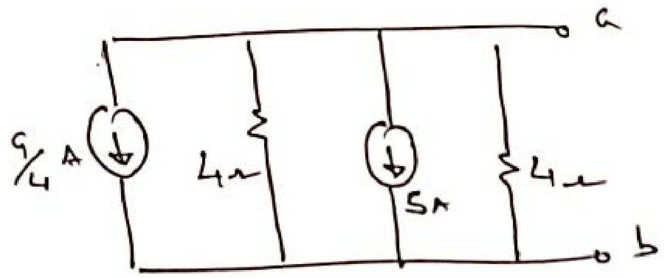


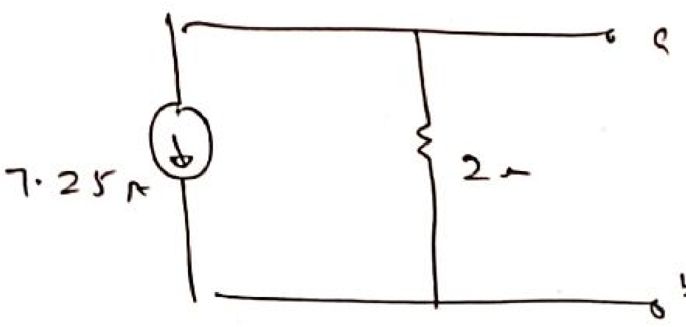

Sol

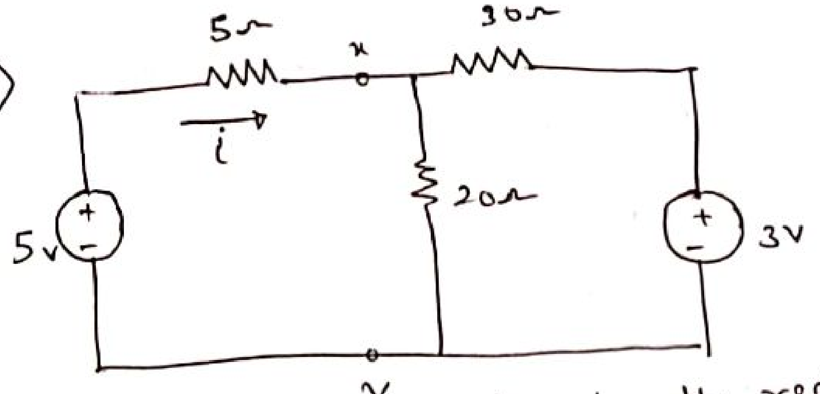



Dummy ∵ it has no effect



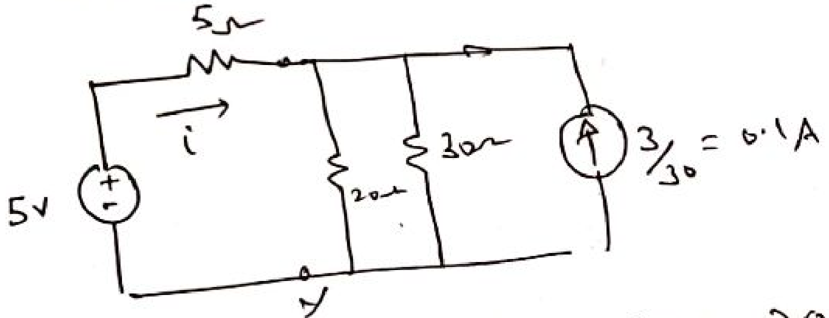



5) 


Find the current $i$, by the reducing the ckt to the right to its simplest form.

Sol/.

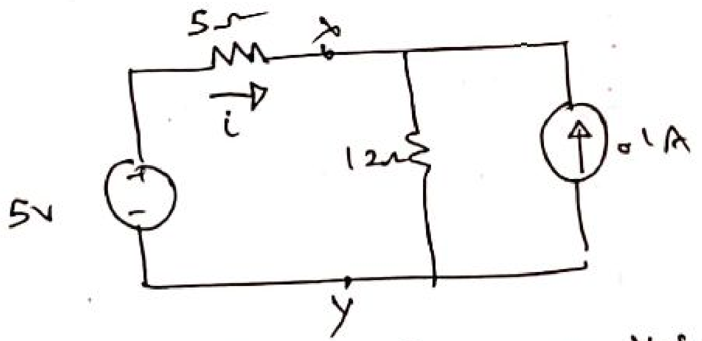
Step 1:- Transform 30Ω resistor 3V src into a current src & a ∥ᶫ resistance



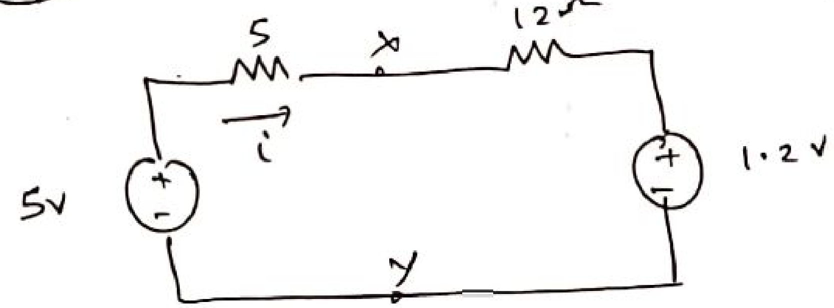

$\frac{3}{30} = 0.1A$

Step 2:- Reducing ∥ᶫ resistors 20 & 30Ω, we have

$$\frac{20 \times 30}{20+30} = 12\Omega$$




Step 3:- Current Src → V.Src



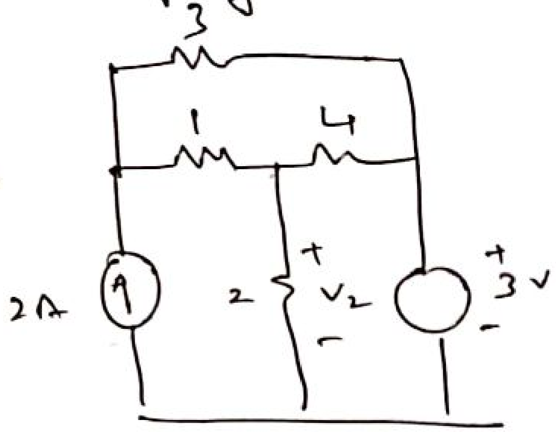

KVL, $5 - 5i - 12i - 1.2 = 0$

$\therefore i = 0.2235A$

Src Shifting

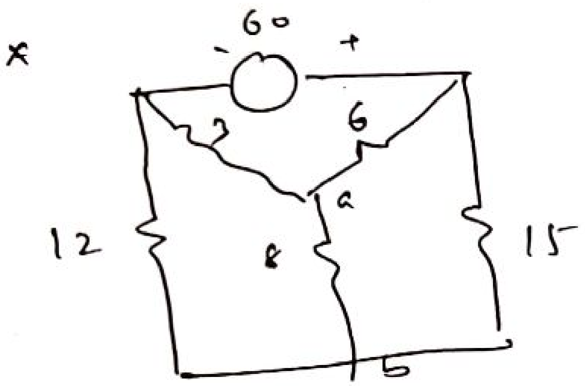
(11)

*




$V_2 = 3V$

*




$V_{ab} = 3.6 V$

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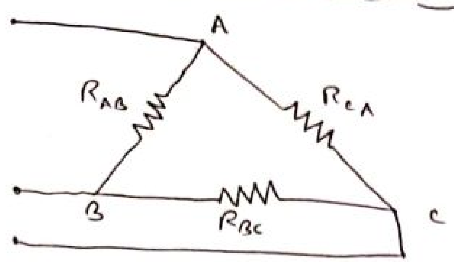
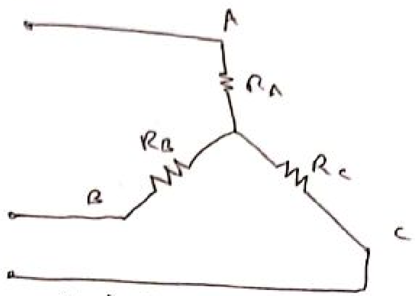
## Star–Delta Transformation



### Delta to Star transformation ($\Delta$ to Y) :-




(a) Delta Connection




(b) Star (or) Y Connection.

Let $R_{AB}$, $R_{BC}$ & $R_{CA}$ be the 3 resistors connected in delta as shown in fig(a) & $R_A$, $R_B$ & $R_C$ be the 3 equivalent resistances connected in Star (Y) as shown in fig (b)

"Principle of Conversion is that the equivalent resistance b/w the corresponding points in both the Connections is Same".

$$\therefore R_A + R_B = R_{AB} \parallel (R_{CA} + R_{BC})$$

$$R_A + R_B = \frac{R_{AB}(R_{CA} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} = \frac{R_{AB}(R_{CA} + R_{BC})}{\Sigma R_{AB}} \quad \text{--- (1)}$$

III$^{ly}$

$$R_B + R_C = \frac{R_{BC}(R_{AB} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} = \frac{R_{BC}(R_{CA} + R_{AB})}{\Sigma R_{AB}} \quad \text{--- (2)}$$

$$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{\Sigma R_{AB}} \quad \text{--- (3)}$$

1 − 2

$$R_A - R_C = \frac{R_{AB}R_{CA} - R_{BC}R_{CA}}{\Sigma R_{AB}} \text{ ——— } (4)$$

3 + 4

$$2R_A = \frac{2R_{AB}R_{CA}}{\Sigma R_{AB}} \qquad \therefore \boxed{R_A = \frac{R_{AB}R_{CA}}{\Sigma R_{AB}}} \text{ — } (5)$$

Ill$^y$

$$\boxed{R_B = \frac{R_{BC}R_{AB}}{\Sigma R_{AB}}} \text{ — } (6) \qquad \boxed{R_C = \frac{R_{BC}R_{CA}}{\Sigma R_{AB}}} \text{ — } (7)$$

Star to $\Delta$ Transformation:

from eq (5), (6) & (7) we can write

$$R_AR_B + R_BR_C + R_CR_A = \frac{R_{AB}R_{BC}R_{CA}(R_{AB}+R_{BC}+R_{CA})}{(\Sigma R_{AB})^2}$$

$$= R_{BC} \times \frac{R_{AB}R_{CA}}{\Sigma R_{AB}}$$

$$= R_{BC} \cdot R_A.$$

$$\therefore \boxed{R_{BC} = R_B + R_C + \frac{R_BR_C}{R_A}}$$

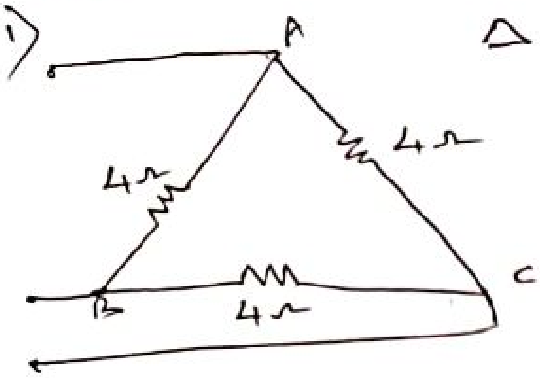
Ill$^y$

$$\boxed{R_{AB} = R_A + R_B + \frac{R_AR_B}{R_C}}$$

$$\boxed{R_{CA} = R_C + R_A + \frac{R_CR_A}{R_B}}$$

Problems

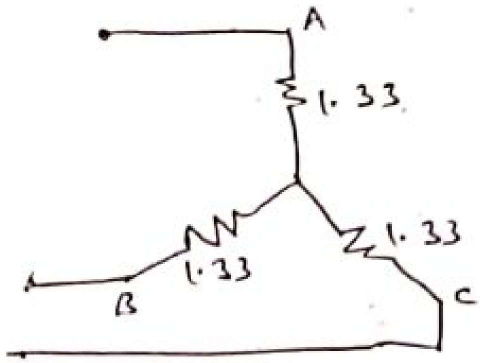
1)




Δ to Y

Sol:

≡

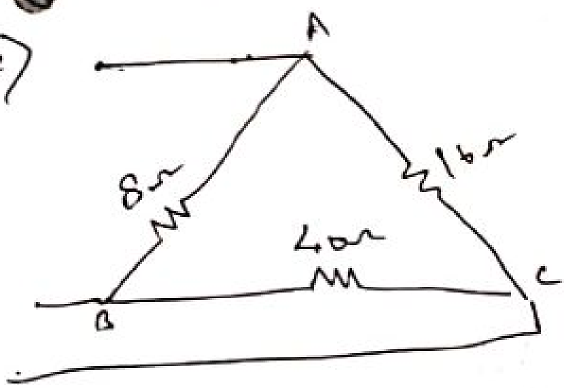



$$R_A = \frac{R_{AB} R_{CA}}{\Sigma R_{AB}} = \frac{4 \times 4}{4+4+4}$$

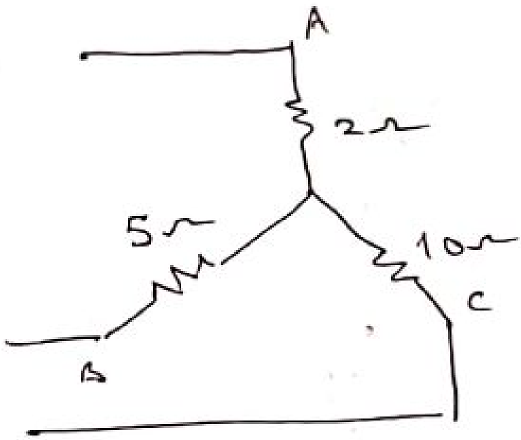
$$= 1.33\,\Omega$$

IIIly $R_B = R_C = 1.33\,\Omega$

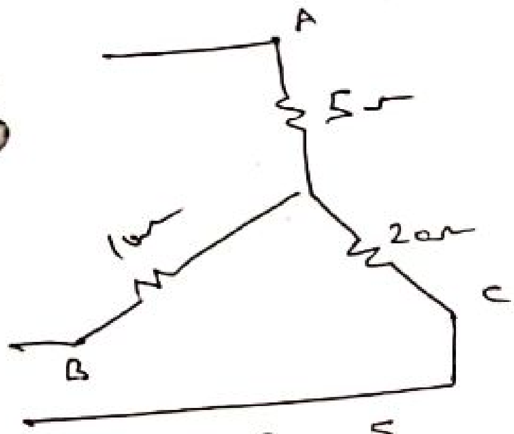
2)



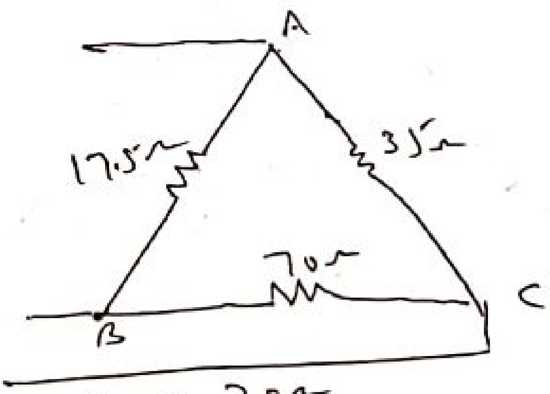

≡




3)

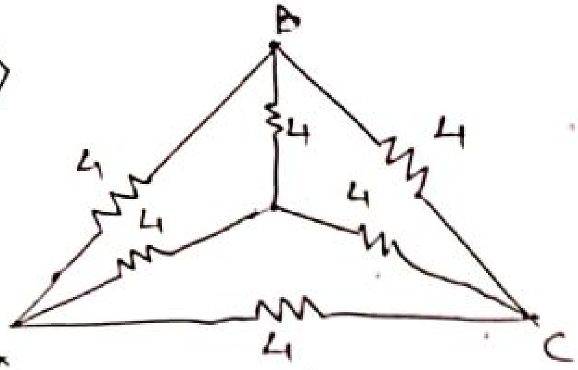



≡



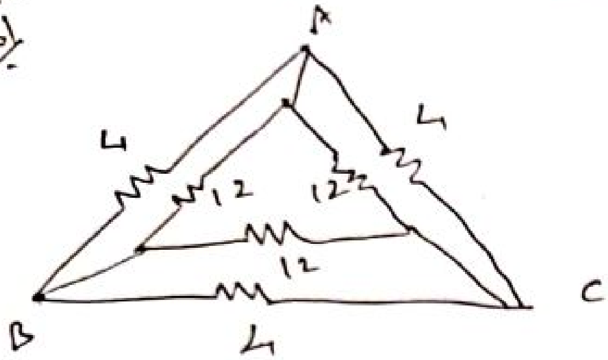

$R_A = 5$ , $R_B = 10$ , $R_C = 20\,\Omega$

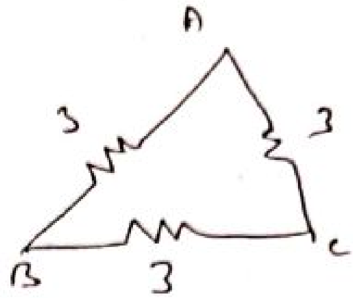
$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

4) 

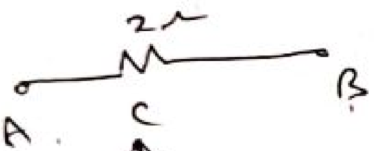
Find the equivalent resistance b/w A & B.

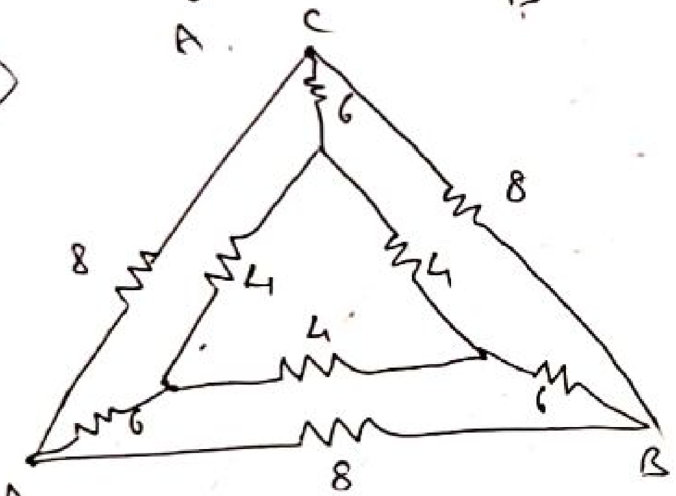
Sol/.





Req. equivalent resistance is $\frac{3 \times 6}{3+6} = 2\,\Omega$



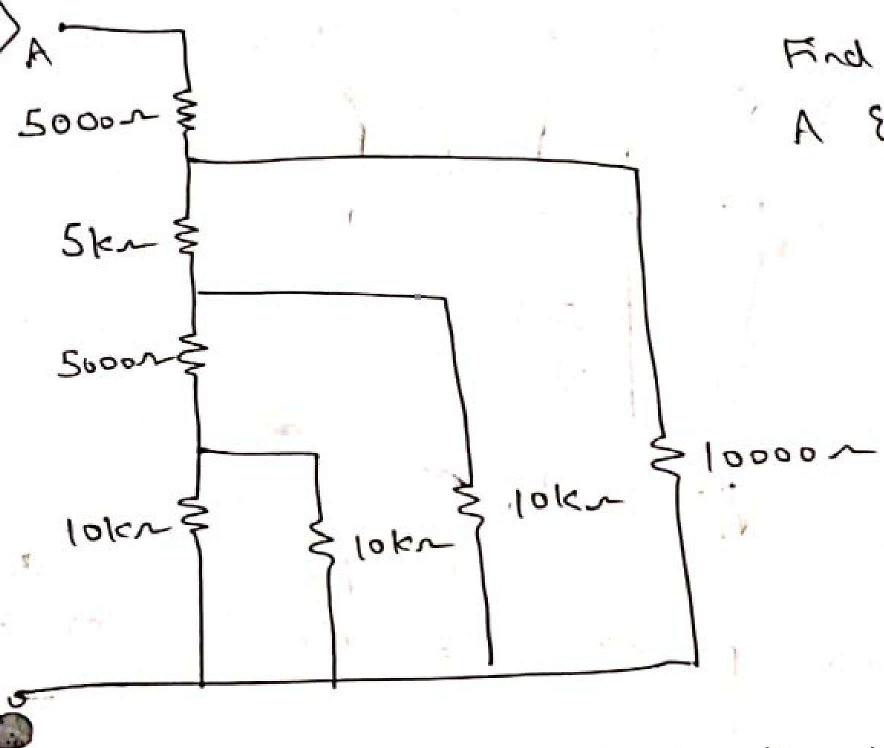
5) 

find the Resistance b/w A & B

Sol/. Ans $3.913\,\Omega$

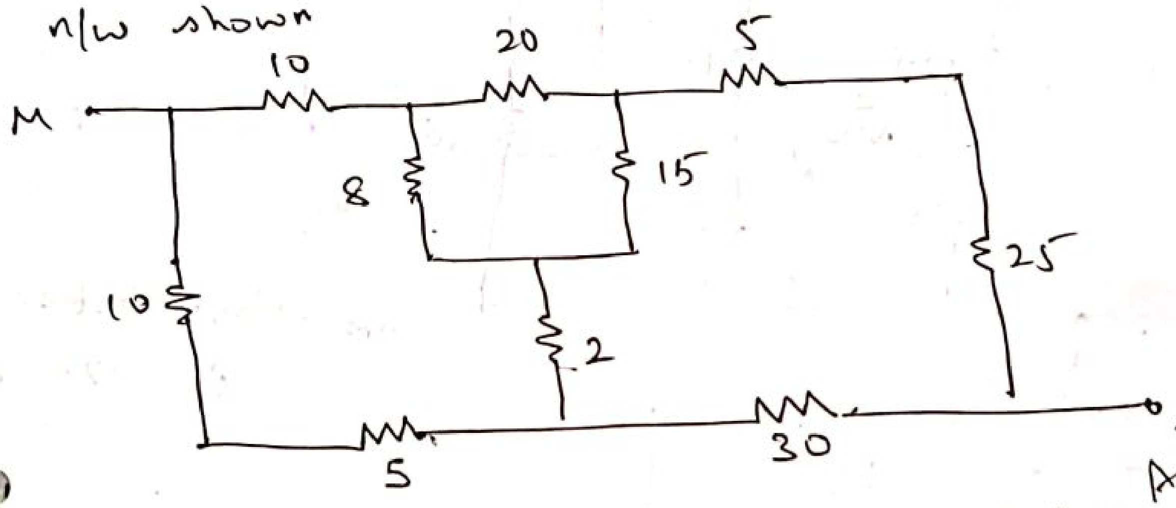
14

6) Find the resistance b/w A & B



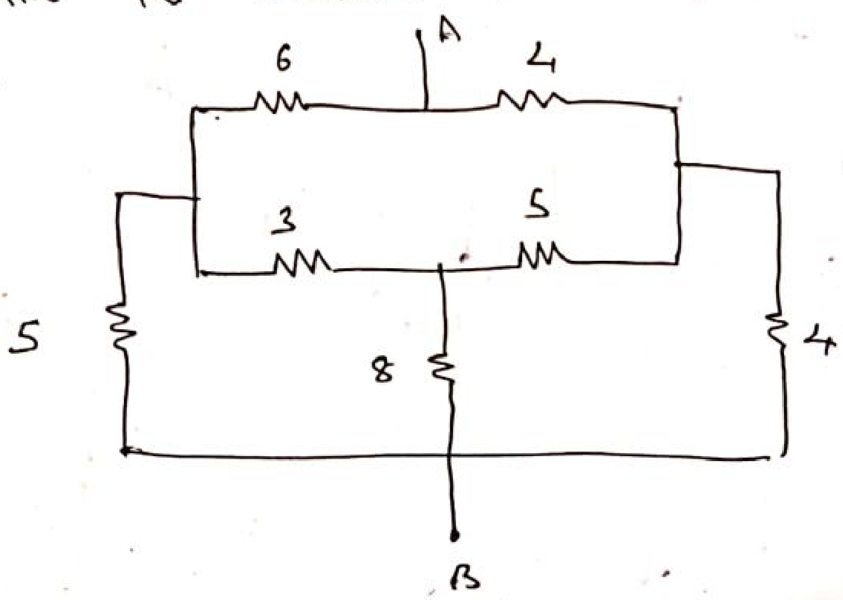

Ans: 10kΩ

7) Determine the resistance b/w the terminals M & N of the n/w shown



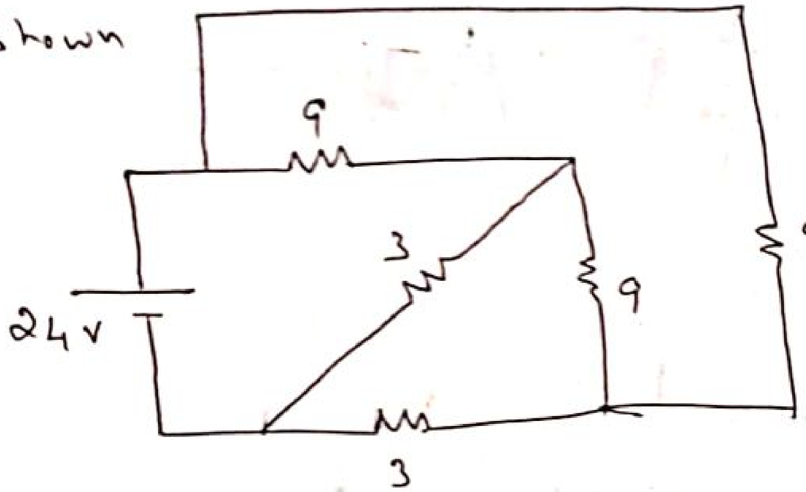

Ans: 23.1Ω

8) Determine the resistance b/w the points A & B in the n/w shown



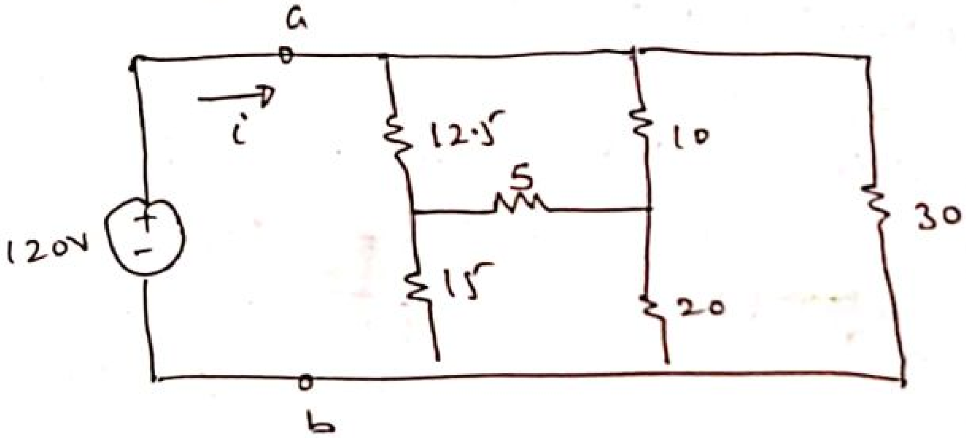

Ans: 4.3Ω

9) Find the current supplied by the src to the n/w shown




Ans: $i = 4A$

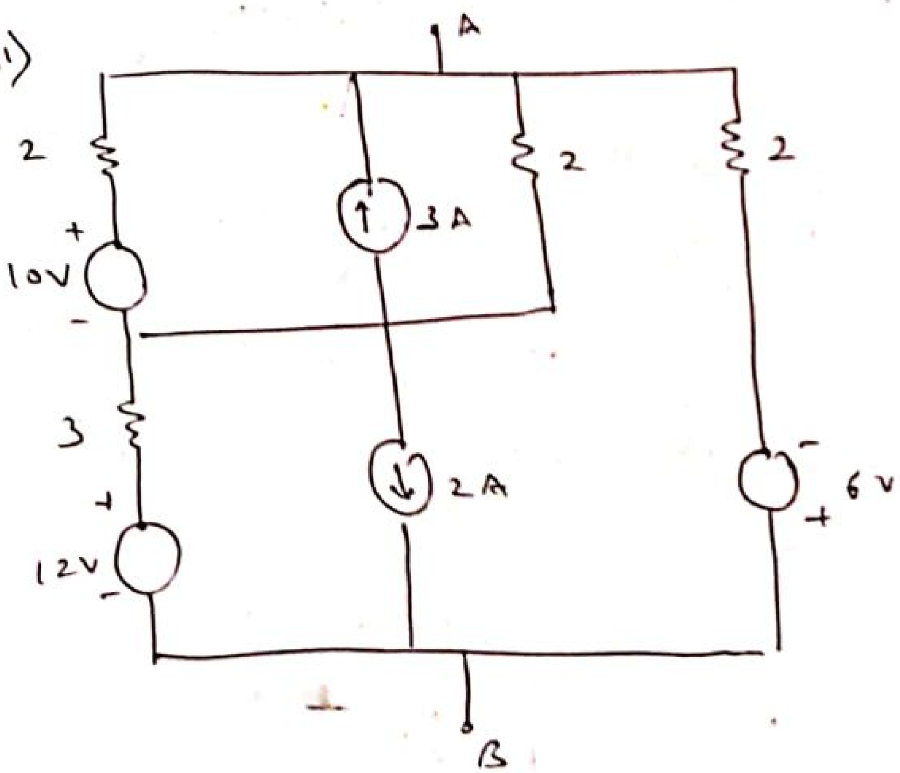
10) Obtain the equivalent resistance $R_{ab}$ for the ckt & hence find i



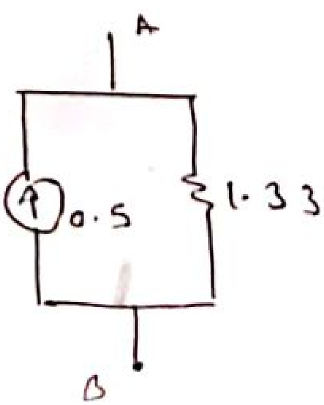

Ans: $R_{ab} = 9.632\,\Omega$

$i = 12.458A$

11)




Ans:

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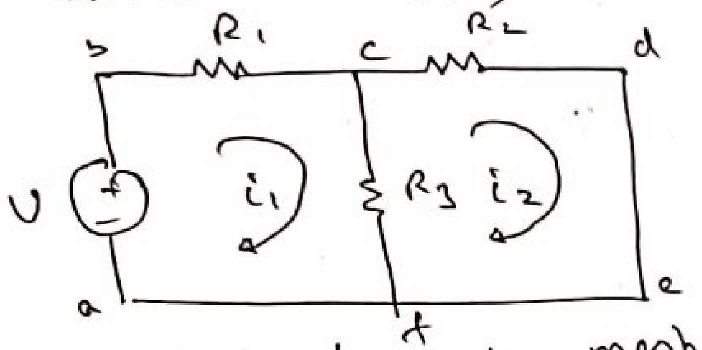
# Mesh Analysis with independent Voltage Src:

Consider the ckt,



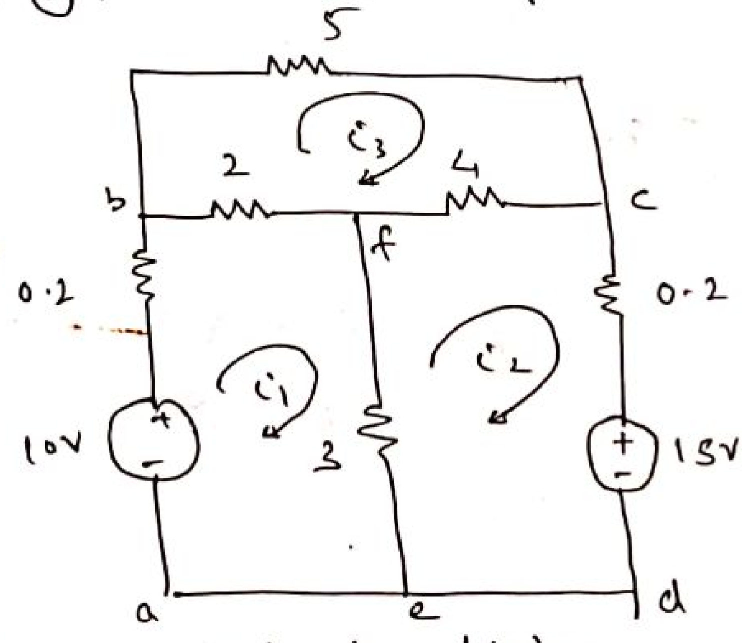

Applying KVL to each mesh,

Mesh1: $V - i_1 R_1 - R_3 (i_1 - i_2) = 0$ —— (1)

Mesh2: $-i_2 R_2 - (i_2 - i_1) R_3 = 0$ —— (2)

Once Mesh Currents are known branch Currents Can be found out.

Eg: (1) Determine the loop Currents & all branch Currents




Applying KVL to Meshes,

Mesh1: $10 - 0.2 i_1 - 2(i_1 - i_3) - 3(i_1 - i_2) = 0$

$5.2 i_1 - 3 i_2 - 2 i_3 = 10$ —— (1)

Mesh 2: $-3(i_2 - i_1) - 4(i_2 - i_3) - 0.2 i_2 - 15 = 0$

$-3 i_1 + 7.2 i_2 - 4 i_3 = -15$ —— (2)

Mesh 3:- $-5i_3 - 4(i_3 - i_2) - 2(i_3 - i_1) = 0$

$$-2i_1 - 4i_2 + 11i_3 = 0 \quad \text{———} (3)$$

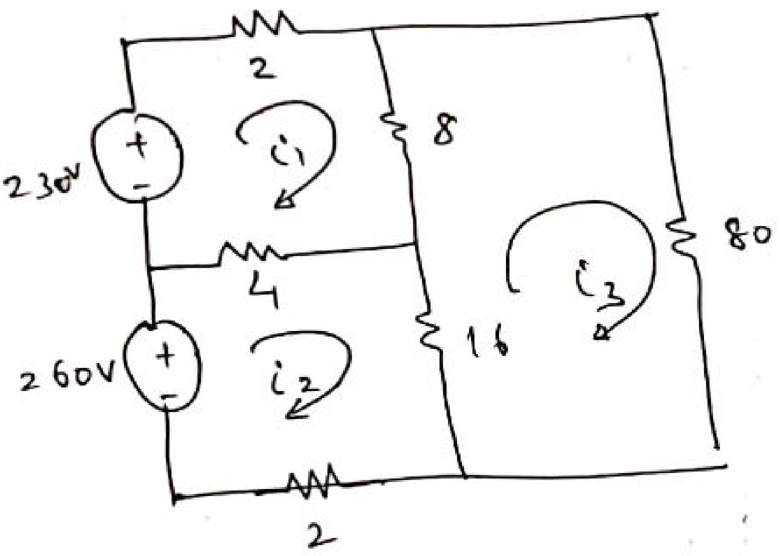
$$\begin{bmatrix} 5.2 & -3 & -2 \\ -3 & 7.2 & -4 \\ -2 & -4 & 11 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -15 \\ 0 \end{bmatrix}$$

using Cramers rule, $i_1 = 0.11A$

$i_2 = -2.53A$

$i_3 = -0.9A$

(2) Find the Power dissipated in the $80\Omega$ resistor using Mesh Analysis

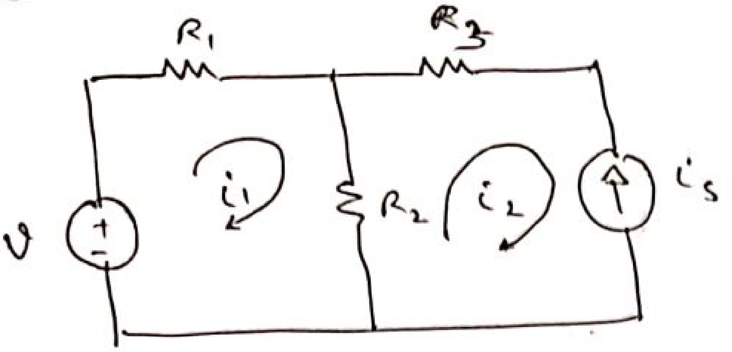



Ans: $i_3 = 5A$

$P_{80} = 2000W$

# Mesh Analysis with independent Current Src

*



Applying KVL to Mesh I,

$$v - i_1 R_1 - R_2(i_1 - i_2) = 0$$

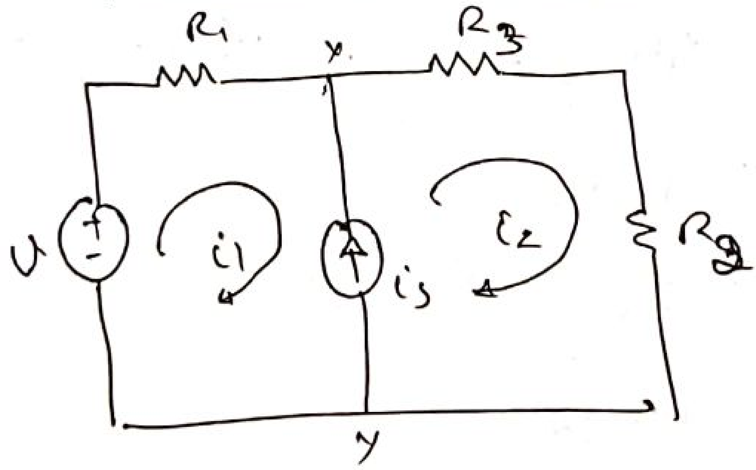
$$i_1(R_1 + R_2) + i_2 R_2 = v \quad \text{---(1)}$$

from Mesh 2, we have $\quad i_2 = -i_s \quad$ ---(2)

$$\therefore \quad i_1(R_1 + R_2) - i_s R_2 = v$$

$$\boxed{i_1 = \frac{v - i_s R_2}{R_1 + R_2}}$$

*



$$i_2 - i_1 = i_s \quad \text{---(1)}$$

M 1:- $\quad v - i_1 R_1 - v_{xy} = 0 \quad$ ---(2)

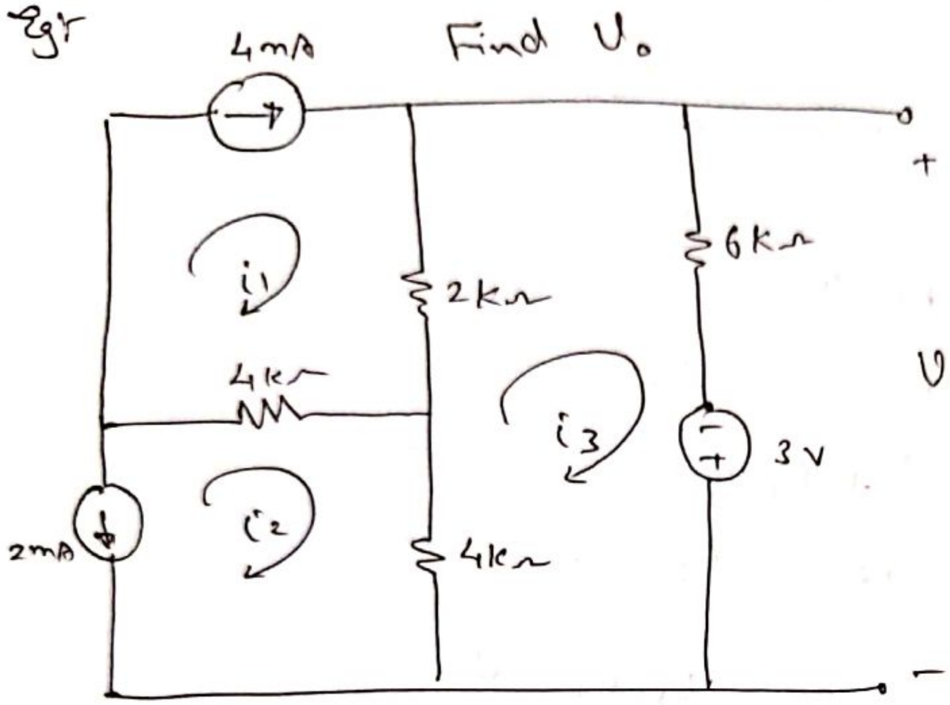
M 2:- $\quad -i_2(R_2 + R_3) + v_{xy} = 0 \quad$ ---(3)

(2)+(3) $\quad i_1 R_1 + (R_2 + R_3) i_2 = v$

Also from eq(1) $\quad i_2 = i_1 + i_s$

$$\therefore \; i_1 = \frac{V - (R_2 + R_3)\, i_s}{R_1 + R_2 + R_3}$$

Eg:- Find $V_o$




$i_1 = 4\,mA$

$i_2 = -2\,mA$

Mesh3:- $-6k\,i_3 - 3 - 4k(i_3 - i_2) - 2k(i_3 - i_1) = 0$

$\therefore \; i_3 = 0.25\,mA$

& $V_o = 6k\,I_3 - 3$

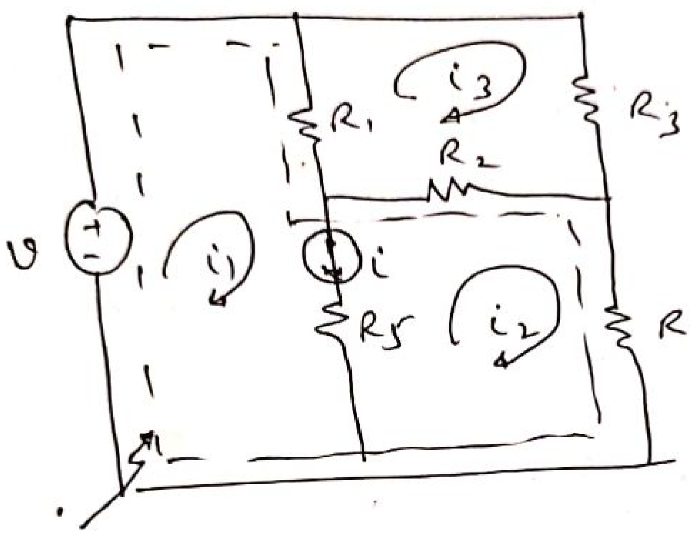
$V_o = -1.5\,V$

## Super Mesh:



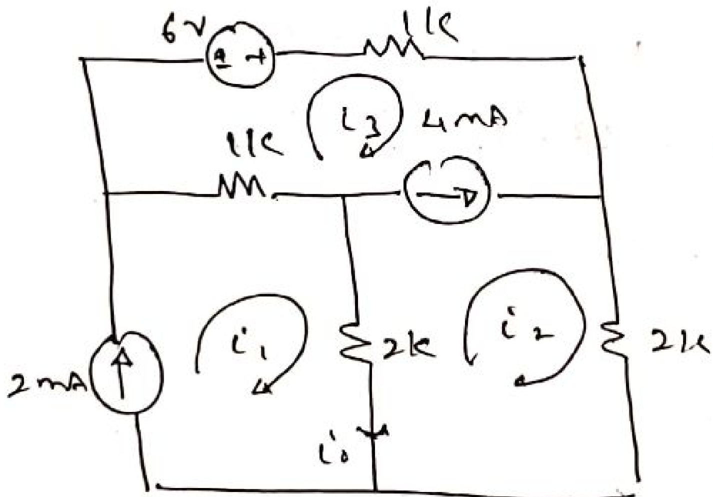
A more General technique for Mesh analysis method, when a Current Src is Common to two meshes, involves the Concept of Super Mesh.

A Super Mesh is Created from 2 meshes that have a Current Src as a Common element, the Current Src is in the interior of a Super Mesh.

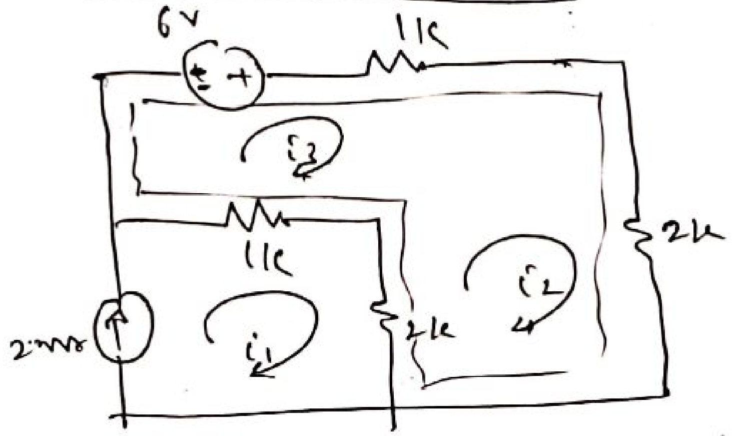



Super Mesh

Eg: Find Current $i_0$




Sol:



M-①, $i_1 = 2mA$, Also $i_2 - i_3 = 4mA$.

SM - ② & ③ $+6 - (1k)i_3 - 2k(i_2) - 2k(i_2 - i_1)$
$-1k(i_3 - i_1) = 0$

$-6 - 1k(i_2 - 4m) - 2k i_2 - 2k(i_2 - 2m)$

$+6 - 1k i_2 + 4 - 4k i_2 + 4 = 0$

$+6 - 1k(i_2 - 4m) - 2k i_2 - 2k(i_2 - 2m)$
$-1k(i_2 - 4m - 2m) = 0$

$+6 - 1k i_2 + 4 - 2k i_2 - 2k i_2 + 4$

$-11k i_2 + 6 = 0$

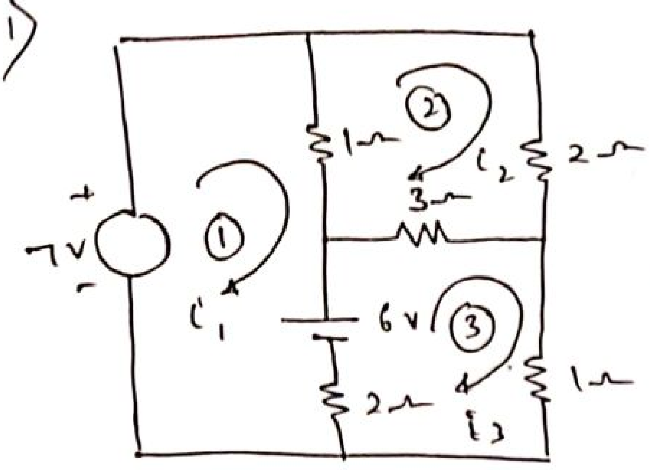
$i_2 = \frac{20}{6} = 3.33mA$

But $i_0 = i_1 - i_2 = 2 - 3.33$

$i_0 = -1.33mA$

Problems

1)




find $i_1, i_2$ & $i_3$.

Sol: Applying KVL to Mesh 1;

$$7 - 1(i_1 - i_2) - 6 - 2(i_1 - i_3) = 0$$

$$3i_1 - i_2 - 2i_3 = 1 \longrightarrow (1)$$

Mesh 2,

$$-i_1 + 6i_2 - 3i_3 = 0 \longrightarrow (2)$$

Mesh 3, $-2i_1 - 3i_2 + 6I_3 = 6 \longrightarrow (3)$

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix} = 3(36-9) + 1(-6-6) - 2(3+12)$$

$$\boxed{\Delta = 39}$$

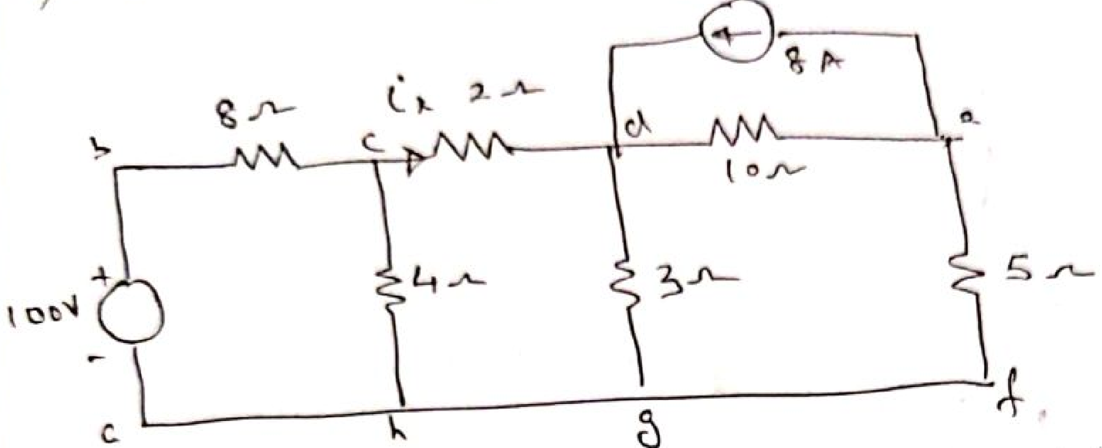
$$\Delta_1 = \begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix} = 1(36-9) + 1(18) - 2(-36)$$

$\Delta_1 = 117$

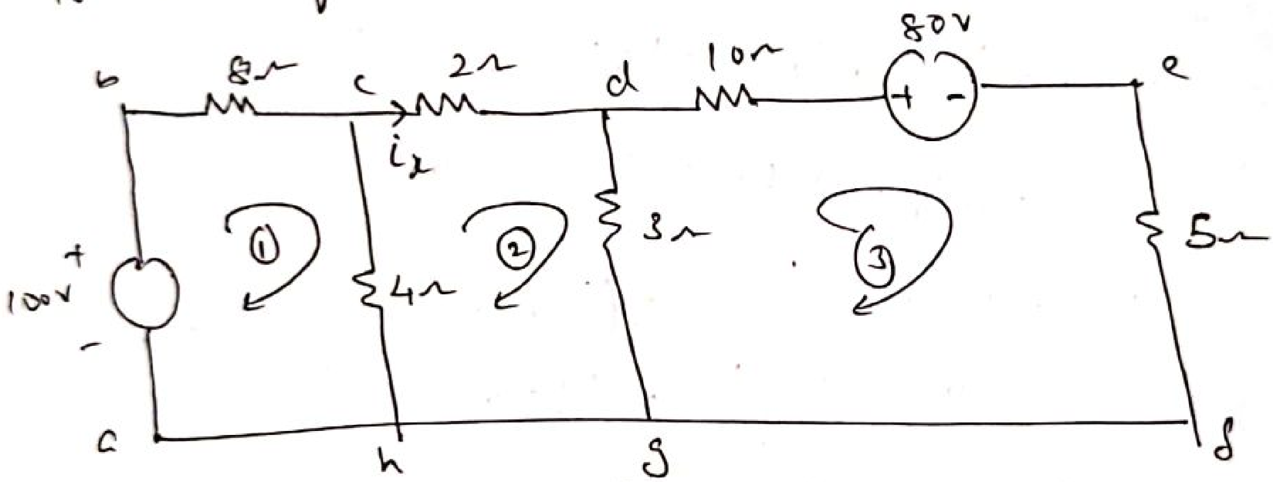
$$\Delta_2 = \begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix} = 78 \qquad \Delta_3 = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix} = 117$$

∴ $i_1 = \frac{\Delta_1}{\Delta} = \frac{117}{39} = 3A$, $i_2 = \frac{\Delta_2}{\Delta} = 2A$, $i_3 = \frac{\Delta_3}{\Delta} = 3A$

2) Use Mesh Analysis to find $i_x$,




Sol/ Convert the the Current Src b/w d & e to its equivalent Voltage Src




Mesh abch, $12i_1 - 4i_2 = 100$

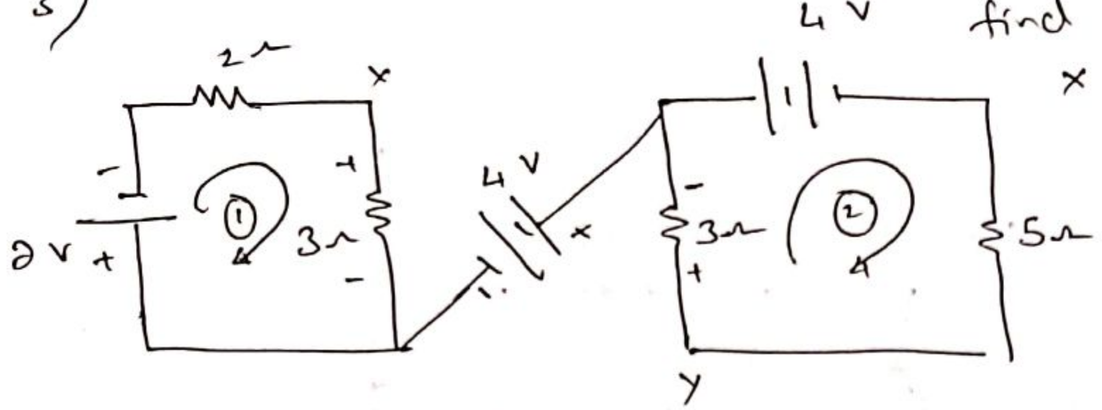
cdgh, $-4i_1 + 9i_2 - 3i_3 = 0$

defg, $-3i_2 + 18i_3 = -80$

$i_2 = i_x = 2.79A$

(19)

3)




find p.d. b/w x & y.

Apply KVL to loop (1),

$-2 - 2i_1 - 3i_1 = 0$

$\therefore -5i_1 = 2$

$i_1 = -0.4A$

Apply KVL to loop (2)

$-3i_2 - 4 - 5i_2 = 0$

$\therefore i_2 = -0.5A$

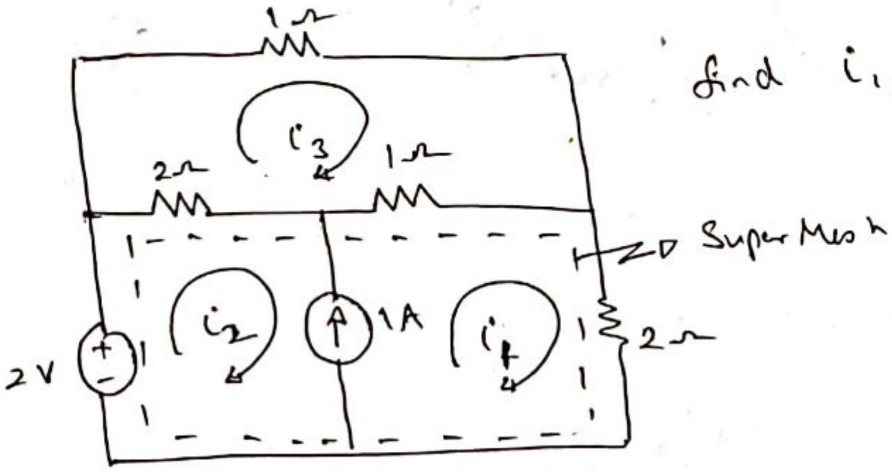

$V_{xy} - 3i_1 + 4 + 3i_2 = 0$

$\cancel{V_{xy} - 3i_1 + 4 + 3i_2 = 0}$

$V_{xy} = \cancel{-3i_1 + 4 + 3i_2}$

$V_{xy} = -3.7V$

4)




find $i_1$, $i_2$ & $i_3$

Super Mesh: $3i_1 + 3i_2 - 3i_3 = 2$

Mesh 3: $2i_1 + i_2 - 4i_3 = 0$

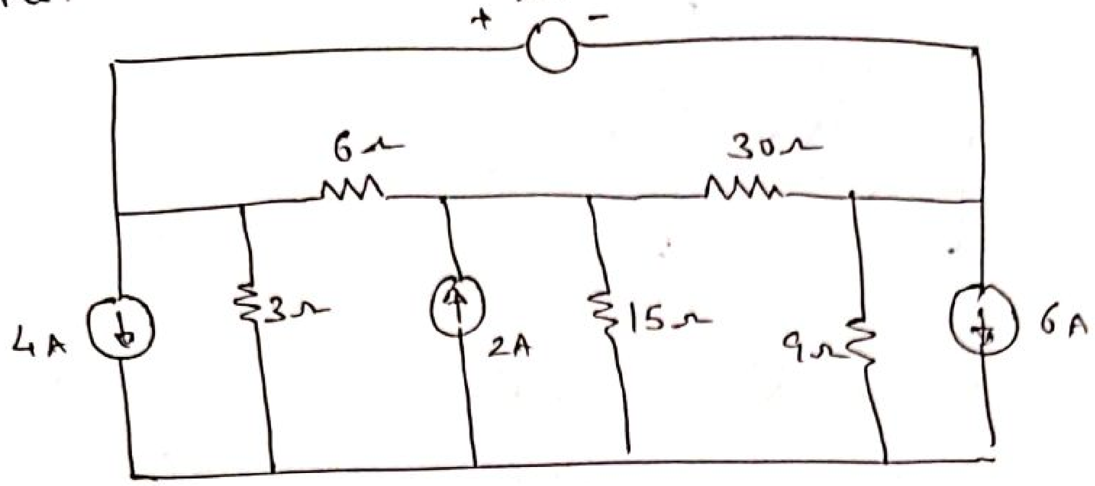
$i_1 - i_2 = 1$

$i_1 = \cancel{1.55A}$ $10/11$

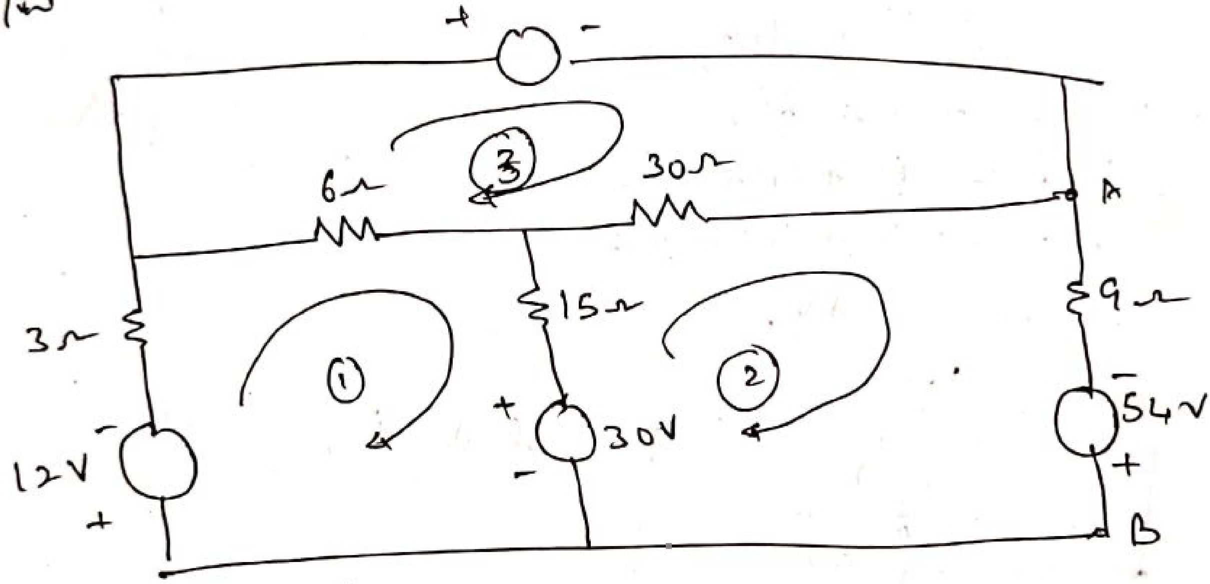
$i_2 = \cancel{0.55A}$ $-1/11$

$i_3 = 0.91A$ $2/11$

5) Using Mesh Current analysis find the voltage across 6A Source.




Sol: Convert all I. src into voltage src & draw the n/w




$$24 i_1 - 15 i_2 - 6 i_3 = -42$$

$$-15 i_1 + 54 i_2 - 30 i_3 = 84$$

$$-6 i_1 - 30 i_2 + 36 i_3 = -20$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{23112}{9612} = 2.4\ A$$

$$V_{AB} = 9 \times i_2 - 54 = -32.4\ V$$

Which is the drop across Current src of 6A

6) The loop equations of a n/w are

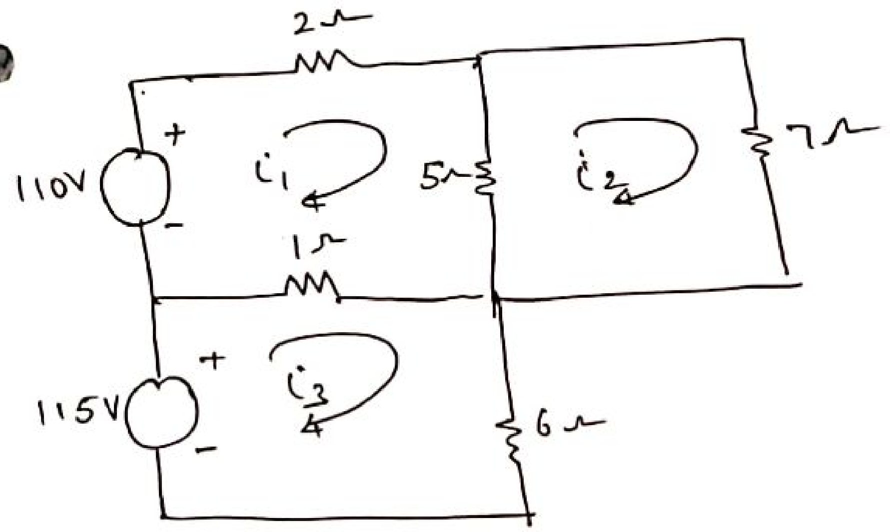
$8I_1 - 5I_2 - I_3 = 110$

$-5I_1 + 12I_2 = 0$

$-I_1 + 7I_3 = 115$, Draw the n/w

Sol: In Matrix form,

$$\begin{bmatrix} 8 & -5 & -1 \\ -5 & +12 & 0 \\ -1 & 0 & 7 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 110 \\ 0 \\ 115 \end{bmatrix}$$

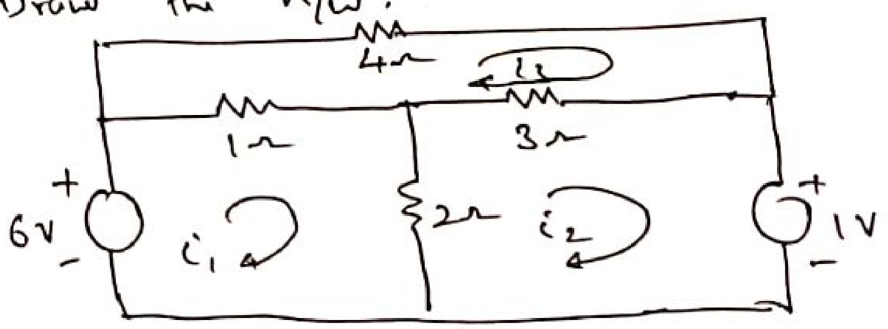



7) The current equations of a certain ckt are

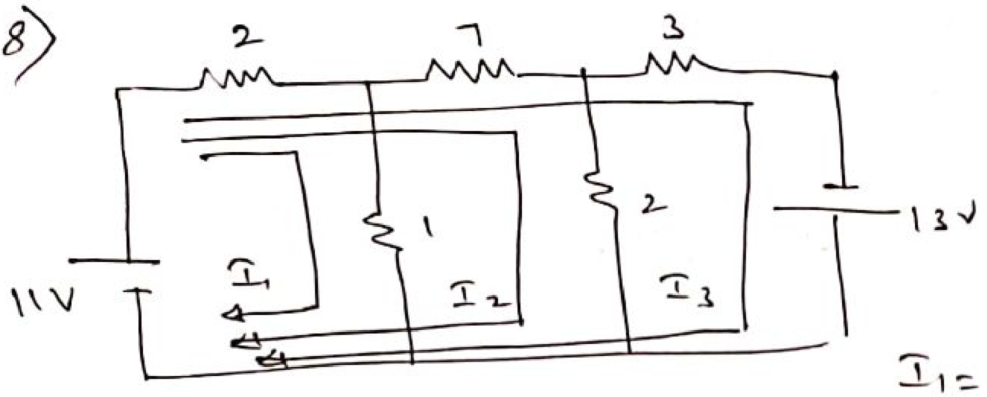
$$\begin{bmatrix} 3 & -2 & -1 \\ -2 & 5 & -3 \\ -1 & -3 & 8 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$$

Draw the n/w.

Sol:



8)




$I_1 = 3A$

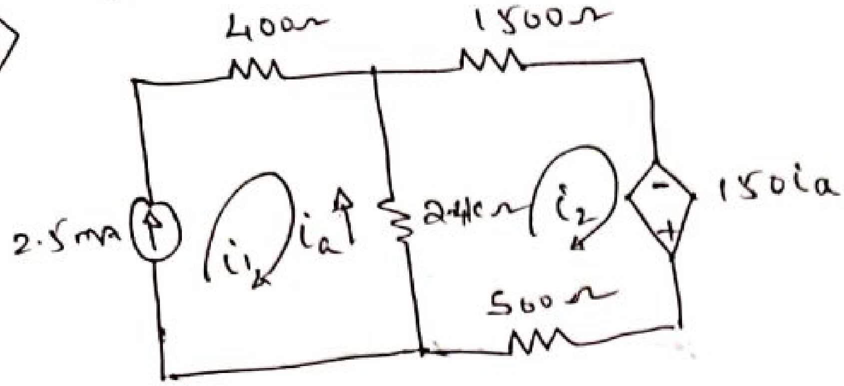
$I_2 = -2A$

$I_3 = 3A$

Scanned with CamScanner

Mesh analysis for the ckts involving dependent srcs.

1)




Sol/ Mesh1:- $i_1 = 2.5$ mA , $i_a = i_2 - i_1$

Mesh 2:- $-2400 i_1 + 4400 i_2 = 150 i_a$

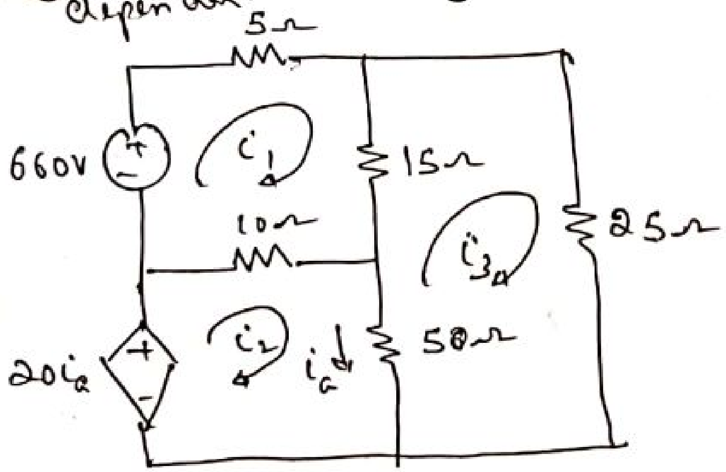
$-2400 i_1 + 4400 i_2 = 150 (i_2 - i_1)$

$5.625 = 4250 i_2$

$i_2 = 1.323$ mA

$\therefore i_a = i_2 - i_1 = -1.176$ mA

2) Find the total power delievered in the ckt by dependent voltage src




Sol/

$30 i_1 - 10 i_2 - 15 i_3 = 660$ —(1)

$-10 i_1 + 60 i_2 - 50 i_3 = 20 i_a = 20 (i_2 - i_3)$

$-10 i_1 + 40 i_2 - 30 i_3 = 0$ —(2)

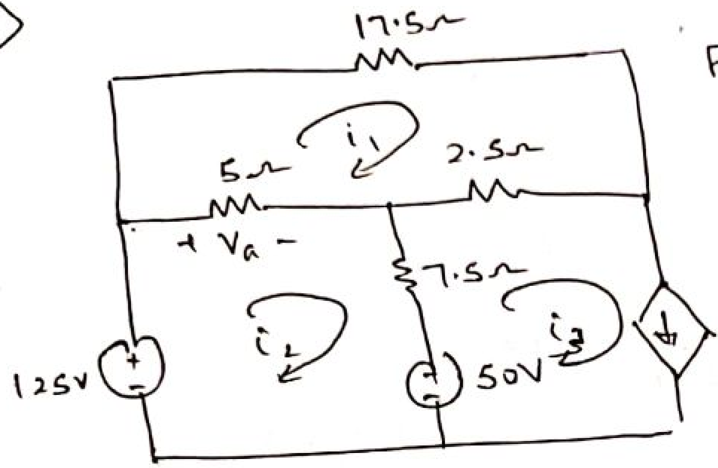
$-15 i_1 - 50 i_2 + 90 i_3 = 0$ —(3)

$i_2 = 27A, \quad i_3 = 22A.$

$i_a = i_2 - i_3 = 5A$

$P_{20i_a} = (20 i_a)\, i_2 = 2700 W$

3)

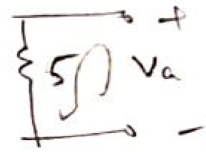



Find the total power delivered in the ckt using Mesh-Current Method

$0.2 V_a$ [delivers]

Mesh 1: $25 i_1 - 5 i_2 - 2.5 i_3 = 0$

2: $-5 i_1 + 12.5 i_2 - 7.5 i_3 = 75$




$i_3 = 0.2 V_a$

$V_a = 5(i_2 - i_1)$ $\quad V_a - 5(i_2 - i_1) = 0$

$\therefore i_3 = 0.2 \times 5(i_2 - i_1) = i_2 - i_1$ $\quad V_a = 5(i_2 - i_1)$

$i_1 = 3.6A, \quad i_2 = 13.2A$

$i_3 = i_2 - i_1 = 9.6A$

$P_{vcs} = 595.2 W$ (absorbed)

$P_{50v} = 180 W$ $\quad$ i.e. $50(i_2 - i_1)$

$P_{125v} = 1650 W.$ $\quad$ i.e. $125 i_2$

(21) b

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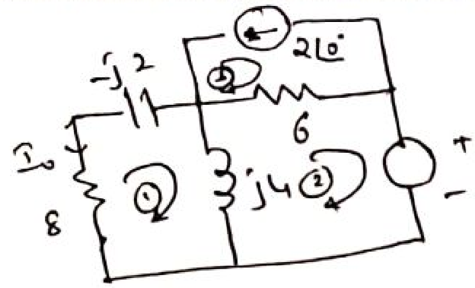


$I_o = ?$

$10\angle 30° = 5\sqrt{3} + j5$

Sol

$I_3 = -2\angle 0° = -2$

$I_3 = -2$ —— (1)

loop (1)

$(8 + j2)I_1 - j4 I_2 = 0$ —— (2)

$-j4 I_1 + (6 + j4)I_2 - 6I_3 = -(5\sqrt{3} + j5)$ —— (3)

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ (8+j2) & -j4 & 0 \\ -j4 & (6+j4) & -6 \end{vmatrix} = 56 + j44$$

$$\Delta_1 = \begin{vmatrix} -2 & 0 & 1 \\ 0 & -j4 & 0 \\ -(5\sqrt{3}+j5) & (6+j4) & -6 \end{vmatrix} = \cancel{56 - j4} \; 20 - j(48 + 20\sqrt{3})$$

$I_1 = \frac{\Delta_1}{\Delta} = -0.496 - j1.085$

$I_1 = 1.193 \angle -114.55$ (or) $1.193 \angle 65.45$ A

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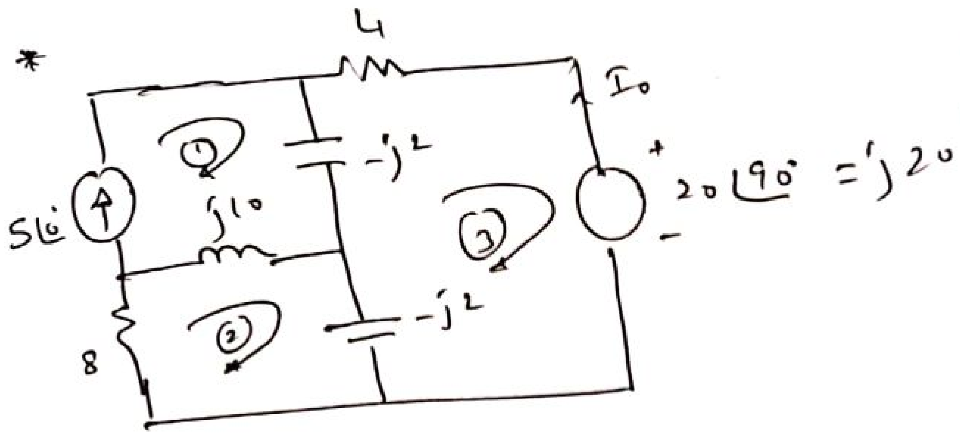



Sol

$$I_1 = +5 \quad \text{--- (1)}$$

$$-j10\, I_1 + (8+j8)\, I_2 + j2\, I_3 = 0 \quad \text{--- (2)}$$

$$j2\, I_1 + j2\, I_2 + (4-j4)\, I_3 = -j20 \quad \text{--- (3)}$$

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ -j10 & (8+j8) & j2 \\ j2 & j2 & (4-j4) \end{vmatrix} = 68$$

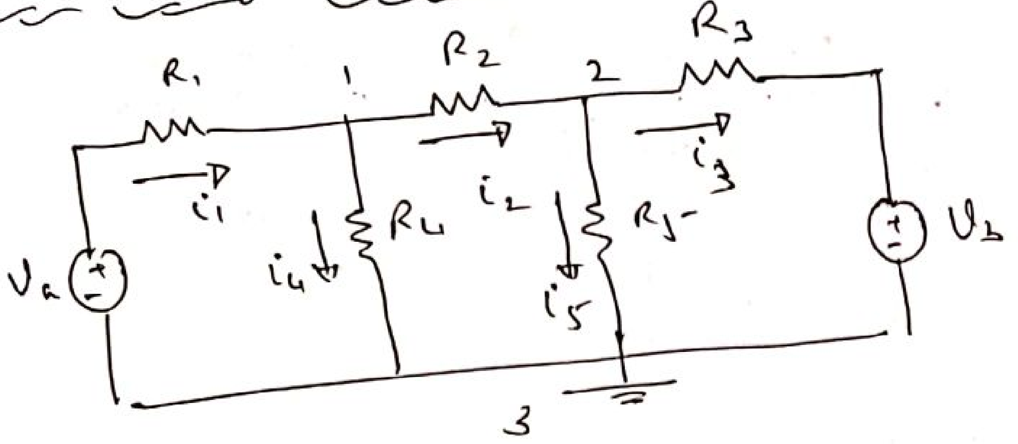
$$\cancel{\Delta_2 = \begin{vmatrix} 1 & -5 & 0 \\ -j10 & 0 & j2 \\ j2 & -j20 & 4-j4 \end{vmatrix} = 180 + j200}$$

$$\Delta_3 = \begin{vmatrix} 1 & 0 & +5 \\ -j10 & (8+j8) & 0 \\ j2 & j2 & -j20 \end{vmatrix} = 340 - j240$$

$$I_3 = \frac{\Delta_3}{\Delta} = 5 - j3.529$$

$$I_o = -I_3 = 6.12 \angle 144° \text{ A}$$

Node voltage Analysis:




Applying KCL at node 1,

$$i_1 = i_2 + i_4$$

$$\frac{v_a - v_1}{R_1} = \frac{v_1 - v_2}{R_2} + \frac{v_1}{R_4}$$

$$v_1\left[\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_4}\right] - \frac{v_2}{R_2} = \frac{v_a}{R_1} \quad \text{—— (i)}$$

at node 2

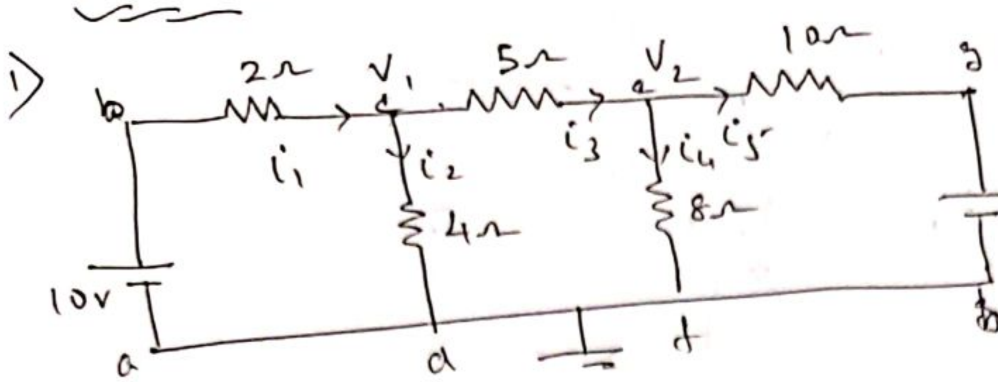
$$i_2 = i_3 + i_5$$

$$\frac{v_1 - v_2}{R_2} = \frac{v_2 - v_b}{R_3} + \frac{v_2}{R_5}$$

$$-\frac{v_1}{R_1} + v_2\left[\frac{1}{R_2} + \frac{1}{R_5} + \frac{1}{R_3}\right] = \frac{v_b}{R_3}$$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} & -\frac{1}{R_2} \\ -\frac{1}{R_1} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_a/R_1 \\ v_b/R_3 \end{bmatrix}$$

Problems

1) 

Find the branch Currents

① $i_1 = i_2 + i_3$

$$\frac{10 - V_1}{2} = \frac{V_1}{4} + \frac{V_1 - V_2}{5}$$

~~$0.95 V_1 - 0.2 V_2 = 5$~~

~~$V_1\left[\frac{1}{4} + \frac{1}{2}\right] + \frac{V_2}{5}$~~

$$V_1\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] - \frac{V_2}{5} = 5$$

② $i_3 = i_4 + i_5$

$$\frac{V_1 - V_2}{5} = \frac{V_2}{8} + \frac{V_2 - 20}{10}$$

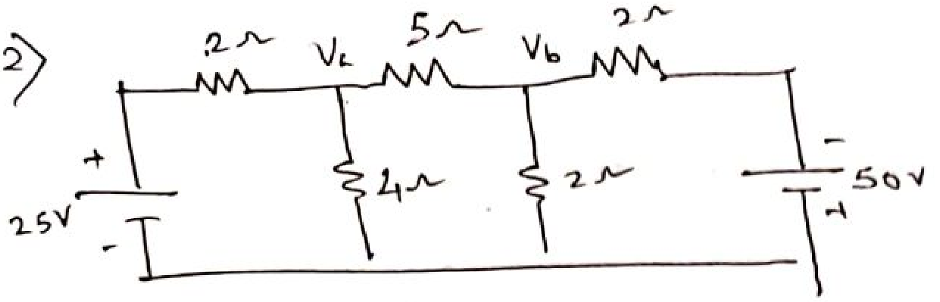
$-0.2 V_1 + 0.425 V_2 = 2$

$V_1 = 6.94 V,\quad V_2 = 7.97 V$

$i_{abc} = 1.53 A \quad , i_{cd} = \quad i_{ce} =$

$i_{ef} = \quad i_{eg}$

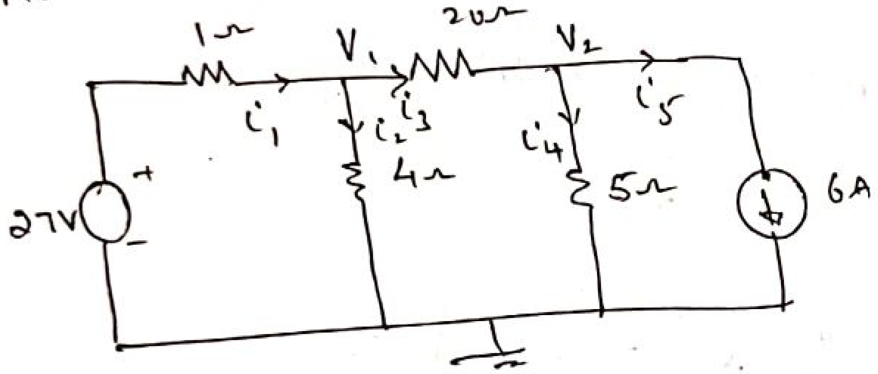
2)




$V_a = 9.09\,V$

$V_b = -19.318\,V$

3) Find the Power dissipated by the 20Ω resistor by node voltage Method




① $i_1 = i_2 + i_3$ —— KCL

$$\frac{27 - V_1}{1} = \frac{V_1}{4} + \frac{V_1 - V_2}{20}$$

$$V_1\left[\frac{1}{4} + \frac{1}{20} + 1\right] - \frac{V_2}{20} = 27 \quad \text{—— (1)}$$

② $i_3 = i_4 + i_5$

$$\frac{V_1 - V_2}{20} = \frac{V_2}{5} + 6$$

$$-V_1\left[\frac{1}{20}\right] + V_2\left[\frac{1}{5} + \frac{1}{20}\right] = -6 \quad \text{—— (2)}$$

$V_1 = 20, \quad V_2 = -20$

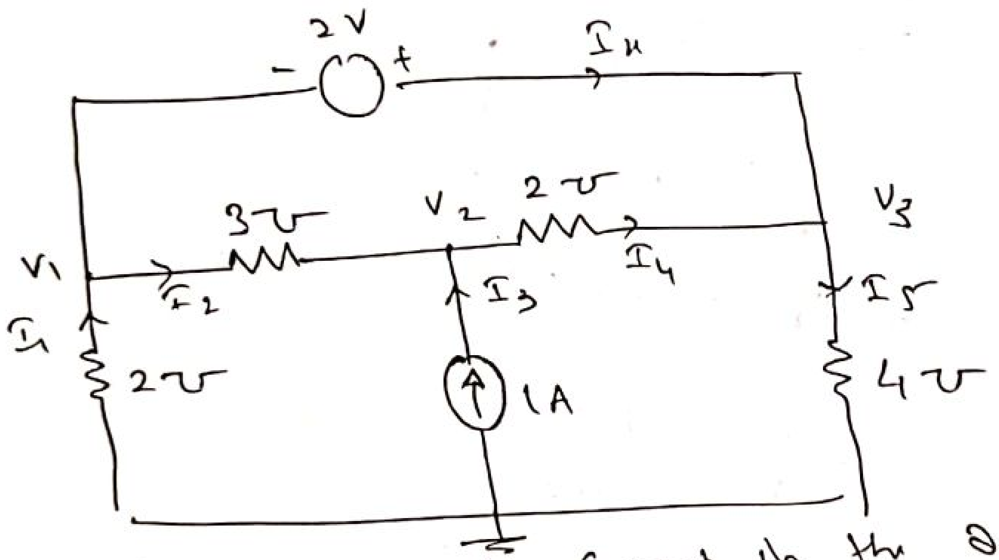
Power dissipated in $20\Omega$ resistor $= \dfrac{V^2}{R}$

$$= \frac{(V_1 - V_2)^2}{R}$$

$$= \frac{(20-(-20))^2}{20}$$

$$= 80 \text{ Watts}$$

4) Determine $V_1$, $V_2$ & $V_3$ by nodal analysis.




Sol: Let $I_x$ be the current in the 2V voltage src branch

node (1) $\quad I_1 = I_2 + I_x$

$$-V_1(2) = (V_1 - V_2)3 + I_x$$

$$5V_1 - 3V_2 + I_x \quad \text{——— (1)}$$

node (2) $\quad I_2 + I_3 = I_4$

$$(V_1 - V_2)3 + 1 = (V_2 - V_3)2$$

$$-3V_1 + 5V_2 - 2V_3 = 1 \quad \text{——— (2)}$$

(3) $I_4 + I_x = I_5$

$(V_2 - V_3)2 + I_x = V_3(4)$

$-2V_2 + 6V_3 - I_x = 0$ —— (3)

from the n/w, $V_3 - V_1 = 2V$ —— (4)

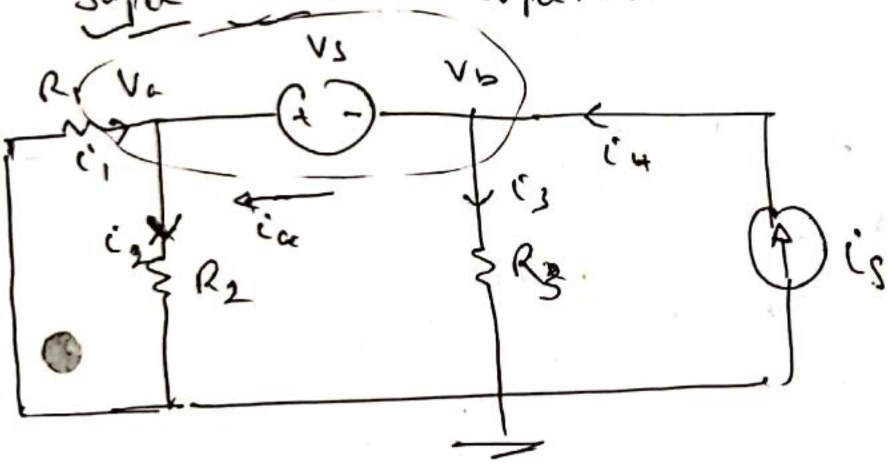
(1) + (2)

$5V_1 - 5V_2 + 6V_3 = 0$ —— (5)

Solving (2), (4), & (5)

$V_1 = -1.166V$, $V_2 = -0.166V$, $V_3 = 0.834V$

Super node: Super node




$V_a - V_b = V_s$ (Constraint eq.) —— (1)

$\cancel{\frac{V_a}{R_1} + i_a = 0}$

(a) $\cancel{i_1 + i_a = i_2}$

$\cancel{\frac{V_a}{R_1} + i_a = \frac{V_a}{R_2}}$ —— ~~(1)~~ $\cancel{\frac{V_a}{R_1} - \frac{V_a}{R_2}} = -i_a$ —— (1

(b) $i_a + i_3 = i_4$

$i_a + V_b/R_3 = i_s$ —— (2)

$\cancel{V_a/R_1} + \cancel{V_b/R_3} = \cancel{V_a/R_2}$

(a) $i_1 + i_a = i_2$

$$\frac{V_a}{R_1} + i_a = \frac{V_a}{R_2}$$

$$\frac{V_a}{R_1} - \frac{V_a}{R_2} + i_a = 0 \quad \text{---} (2)$$

(b) $i_a + i_3 = i_4$

$$i_a + \frac{V_b}{R_3} = i_s \quad \text{---} (3)$$

(2) - (3)

$$\frac{V_a}{R_1} - \frac{V_a}{R_2} - \frac{V_b}{R_3} = - i_s \quad \text{---} (4)$$

Solving (1) & (4), $V_a =$ , $V_b =$

[OR]

$$i_1 + i_4 = i_2 + i_3$$

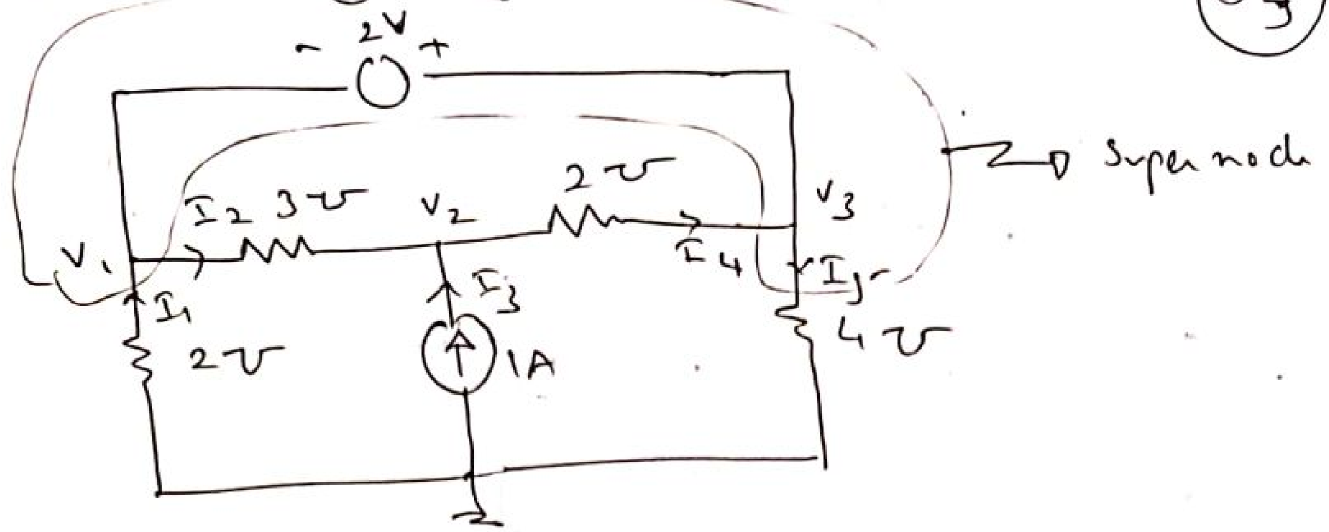
$$\frac{V_a}{R_1} + i_s = \frac{V_a}{R_2} + \frac{V_b}{R_3}$$

$$\frac{V_a}{R_1} - \frac{V_a}{R_2} - \frac{V_b}{R_3} = -i_s \quad \text{---} (5)$$

(4) & (5) are |||

Solve Prob (4) using Super node Method

(24)




Supernode (1) & (3)

$$I_1 + I_4 = I_2 + I_5$$

$$-V_1(2) + (V_2 - V_3)2 = (V_1 - V_2)3 + V_3(4)$$

$$5V_1 - 5V_2 + 6V_3 = 0 \quad \text{—— (1)}$$

Also

$$V_3 - V_1 = 2 \quad \text{—— (2)}$$

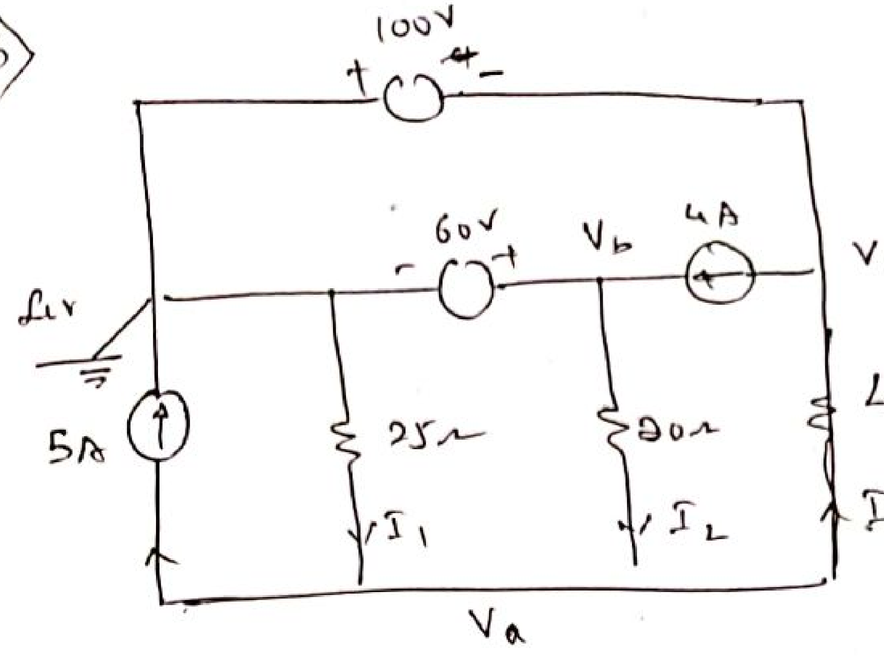
(2) $$I_2 + I_3 = I_4$$

$$(V_1 - V_2)(3) + 1 = (V_2 - V_3)2$$

$$-3V_1 + 5V_2 - 2V_3 = 1 \quad \text{—— (3)}$$

$$V_1 = -1.166V, \quad V_2 = -0.166V, \quad V_3 = 0.834V$$

5)




find the voltage across $40\Omega$ & power supplied by 5A src.

Sol: $V_c = -100V$, $V_b = 60V$.

(a)

$$5 + I_3 = I_1 + I_2$$

$$5 + \frac{V_a - V_c}{40} = \frac{-V_a}{25} + \frac{V_b - V_a}{20}$$

$$5 + \frac{V_a - 100}{40} = -\frac{V_a}{25} + \frac{60 - V_a}{20}$$

$$V_a\left[\frac{1}{25} + \frac{1}{20} + \frac{1}{40}\right] + 5 - \frac{60}{20} + \frac{100}{40} = 0$$

$$V_a = -39.13V$$

$$V_{40\Omega} = V_a - V_c$$
$$= -39.13 + 100$$
$$= 60.86V$$

$$P_{5A\,src} = |V_a| \times 5 = 195.65W$$

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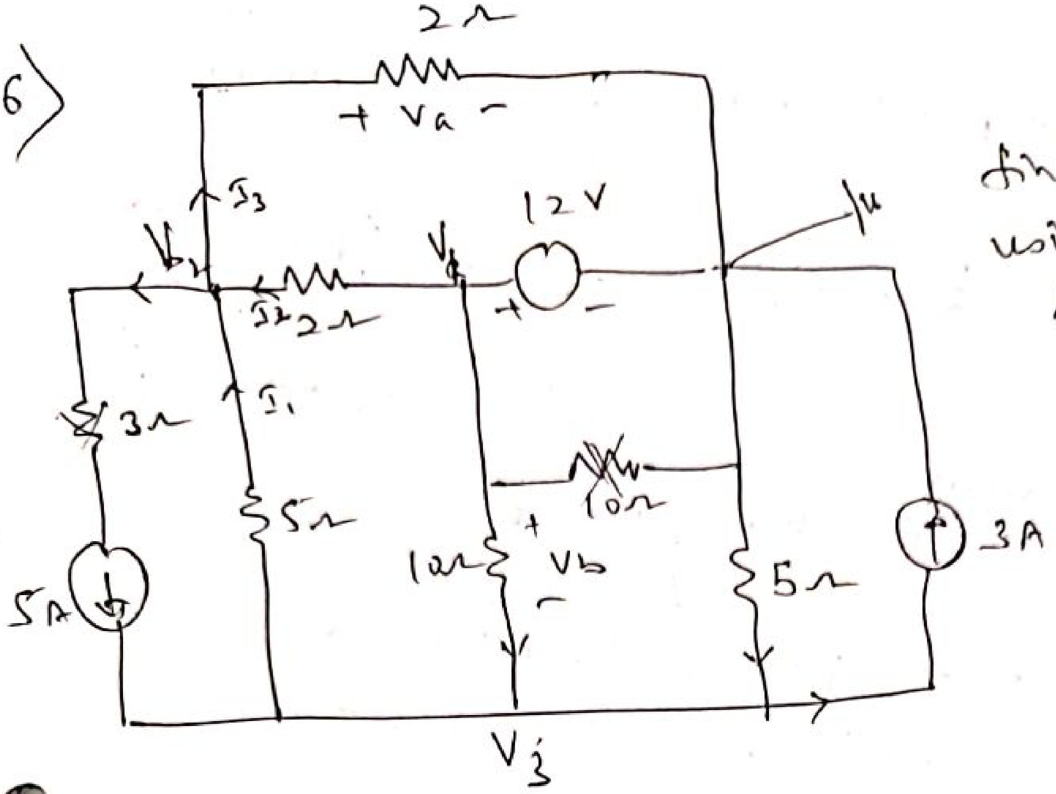
(26)

6)




find Va & Vb using node voltage Method

30% $V_1 = 12V$

@-(2)

$$\frac{V_1 - V_2}{2} + \frac{V_3 - V_2}{5} = \frac{V_2}{2} + 5$$

$$V_1\left[\tfrac{1}{2}\right] - V_2\left[\tfrac{1}{2} + \tfrac{1}{5} + \tfrac{1}{2}\right] + V_3\left[\tfrac{1}{5}\right] = 5$$

$0.5V_1 - 1.2V_2 + 0.2V_3 = 5$

$0.5V_1 - \cancel{1.2V_2 = 5}$ ~~— (1)~~ $\therefore V_2 = \cancel{0.833V} = V_a$ ~~But $V_1 = 12V$~~

$-1.2V_2 + 0.2V_3 = -1$ —— (1)

@-(3)

$$5 + \frac{V_1 - V_3}{10} - \frac{V_3}{5} = 3 + \frac{V_3 - V_2}{5}$$

$$V_1\left[\tfrac{1}{10}\right] + \frac{V_2}{5} - V_3\left[\tfrac{1}{10} + \tfrac{1}{5} + \tfrac{1}{5}\right] = 3 - 5$$

$0.1V_1 + 0.2V_2 - 0.5V_3 = -2$

$0.2V_2 - 0.5V_3 = -3.2$ —— (2)

$V_2 = 2.035V$, $V_3 = 7.21V$

$V_a = V_2 = 2.035V$, $V_b = V_1 - V_3 = 4.79V$

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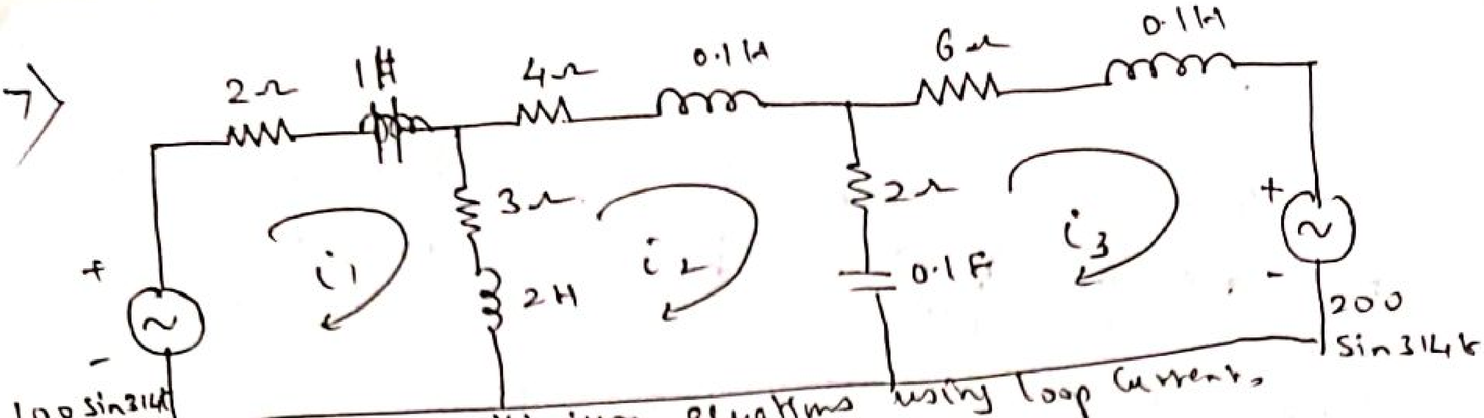
7)




Obtain the equilibrium equations using loop currents.

Sol/ $V_1 = V_m \sin \omega t$

$V_m = 100$, $\omega = 314 = 2\pi f$

$\therefore f = 50\,Hz$.

$Z_1 = R_1 + jX_{L_1} = 2 + j\omega L_1 = (2 + j314)\Omega$

$Z_2 = R_2 + jX_{L_2} = 3 + j\omega L_2 = (3 + j628)\Omega$.

$Z_3 = R_3 + jX_{L_3} = 4 + j\omega L_3 = 4 + j31.4\Omega$

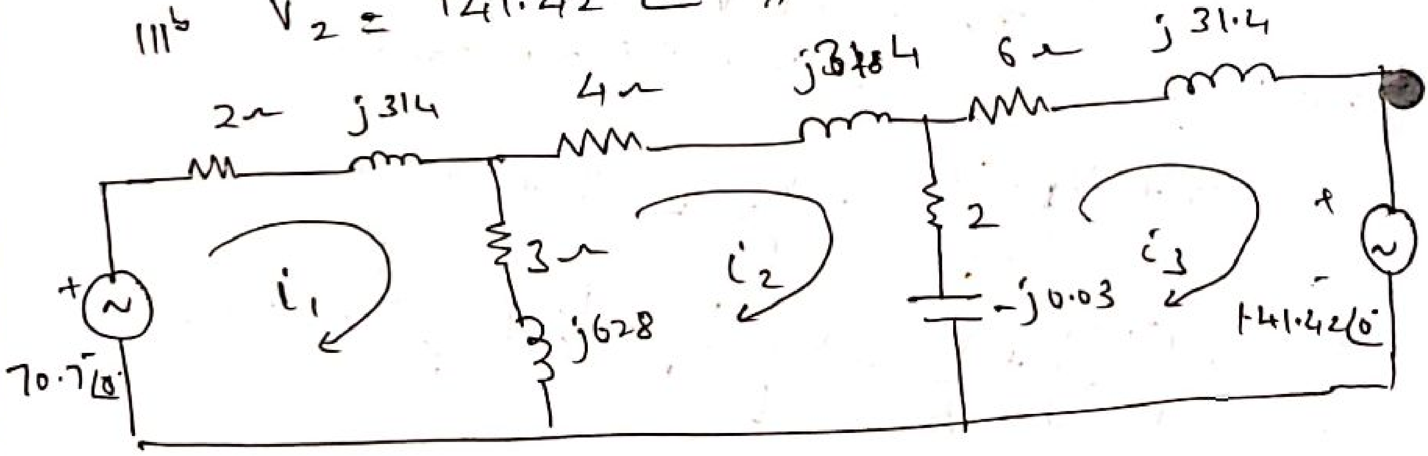
$Z_4 = R_4 - jX_C = 2 - j\frac{1}{\omega C} = 2 - j0.03\Omega$

$Z_5 = 6 + jX_{L_4} = 6 + j31.4\Omega$

R.M.S. value of $V_1 = \frac{100}{\sqrt{2}} = 70.7V$

$\therefore V_1 = 70.7\angle 0°\ V$

$|||^{ly}$ $V_2 = 141.42\angle 0°\ V$



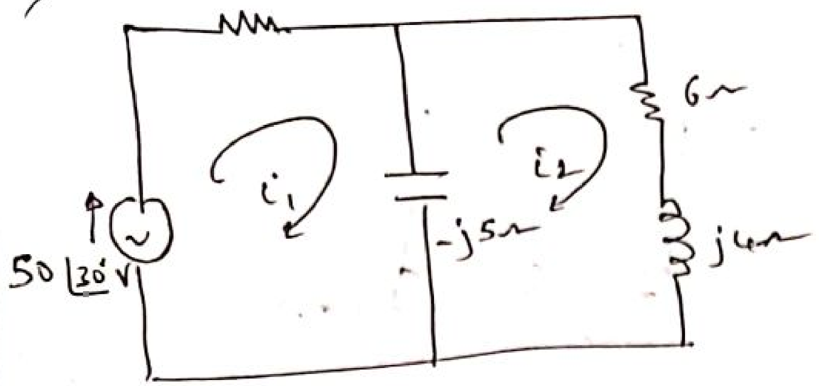

loop (1) KVL

$(5 + j942)\,i_1 - (3 + j628)\,i_2 = 70.7\angle 0°$ —(1)

loop (2) $-(3 + j628)\,i_1 + (9 + j659.4)\,i_2 - (2 - j0.03)\,i_3 = 0$ —(2)

loop (3) $-(2 - j0.03)\,i_2 + (8 + j31.39)\,i_3 = -141.42\angle 0°$ —(3)

8)




$50\angle 30° = 43.3 + j25$

loop ① $(8 - j5)I_1 + j5I_2 = 50\angle 30°$ ——— (1)

loop ② $j5I_1 + (6 - j)I_2 = 0$ ——— (2)

$$\Delta = \begin{vmatrix} 8 - j5 & j5 \\ j5 & 6 - j \end{vmatrix} = 68 - j38$$

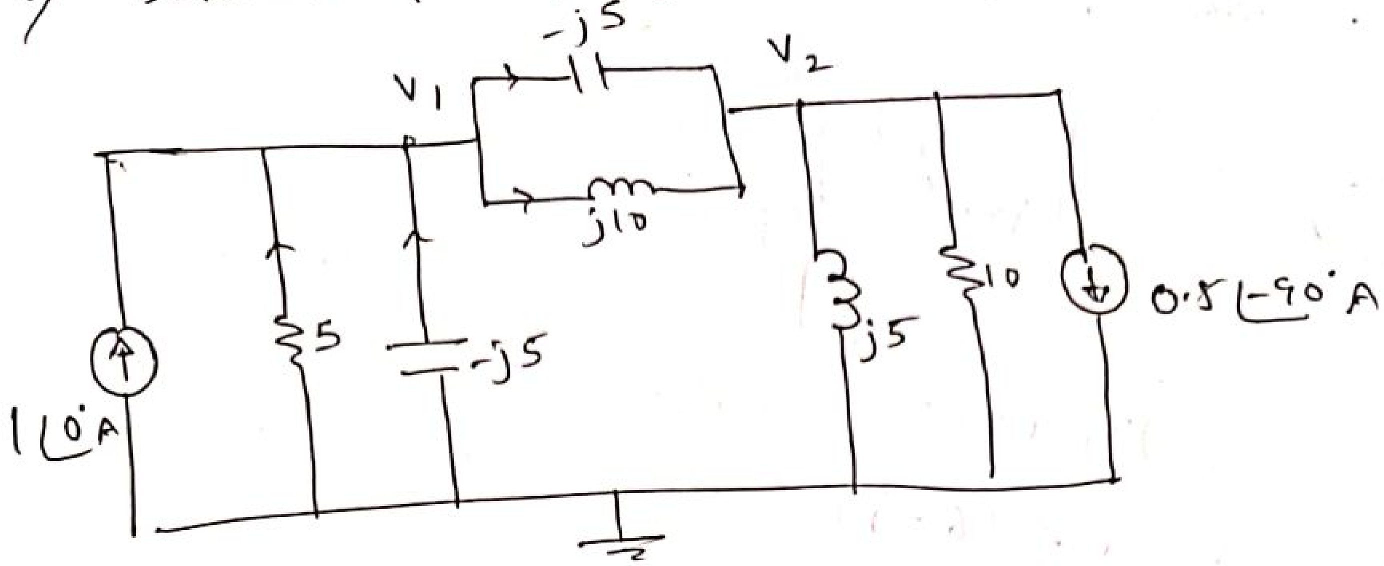
$$\Delta_1 = \begin{vmatrix} 43.3 + j25 & j5 \\ 0 & 6 - j \end{vmatrix} = 284.8 + j106.78$$

$$I_1 = \frac{\Delta_1}{\Delta} = 2.52 + j\,2.979 = 3.9\angle 49.7 \text{ A}$$

$$I_2 = 3.2\angle -30.8 \text{ A}$$

1

9) Determine $V_1$ & $V_2$ by nodal analysis




Sol/ @ node ①

$$-\frac{V_1}{5} + \frac{V_1}{j5} + 1 = \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10}$$

$$V_1\left[\frac{1}{j10} + \frac{1}{5} - \frac{1}{j5} - \frac{1}{j5}\right] - V_2\left[-\frac{1}{j5} + \frac{1}{j10}\right] = 1$$

$$V_1(0.2 + j0.3) - V_2(j0.1) = 1 \quad \text{--- (1)}$$

@ node 2,

$$-V_1(j0.1) + V_2(0.1 - j0.1) = -0.5\angle{-90°} \quad \text{--- (2)}$$

$V_1 = 1.833\angle{-73°}\ V$

$V_2 = 4.136\angle{242.8°}\ V$

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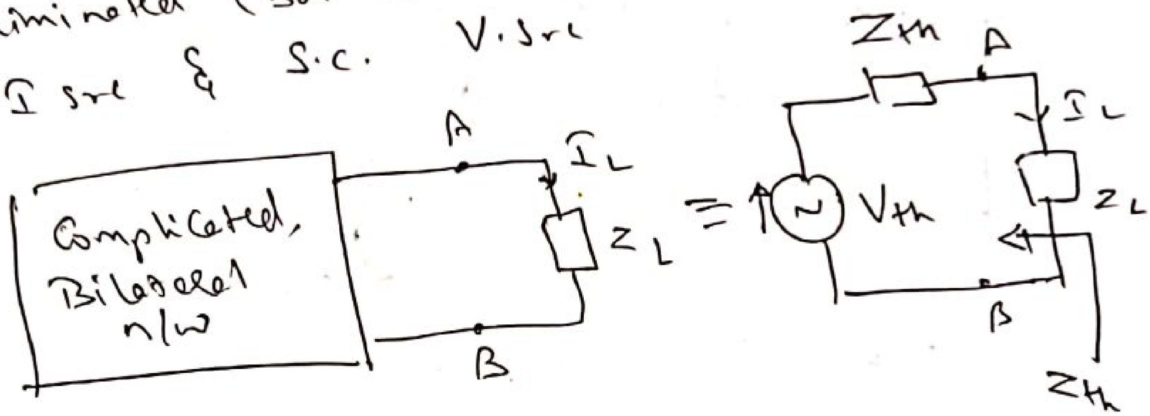
Thevenin's Theorem :-

" Any 2 terminal linear n/w containing energy sources (generators) & impedances can be replaced with an equivalent ckt (containing) consisting of a voltage source $V_{th}$ in series with an impedance $Z_{th}$".

The value of $V_{th}$ is the O.C voltage b/w the terminals of the n/w & $Z_{th}$ is the impedance measured b/w the terminals of the n/w with all energy sources eliminated (but not their impedances)

i.e O.C. I src & S.C. V.src




$$I_L = \frac{V_{th}}{Z_{th} + Z_L}$$

$Z_{th}$ → Thevenin's Equivalent impedance

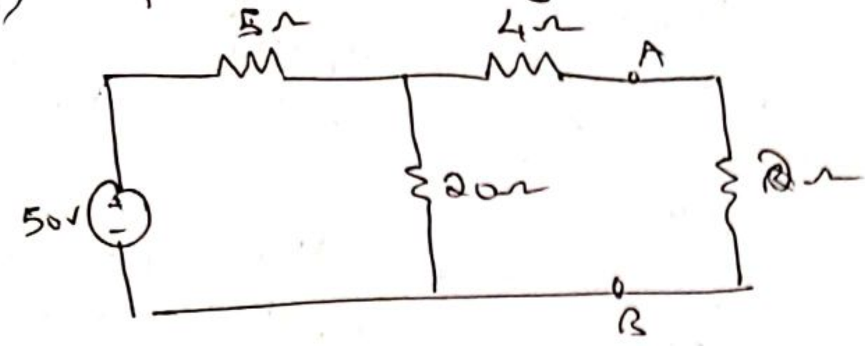
$V_{oc}$ (or) $V_{th}$ → Thevenin's " voltage.

Note: * To find $Z_{th}$, O.C — I src & S.C.-Vsrc Remove the load impedance & find the resistance ~~looking~~ as looked from the terminals of load

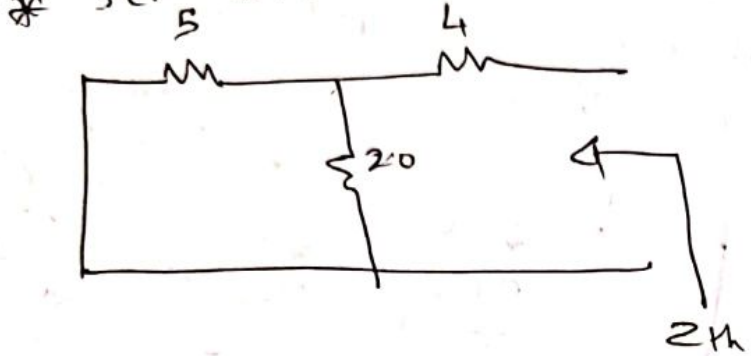
* To find $V_{th}$, O.C. the load, & find O.C. voltage across the terminals

Problems:-

1) find $i_{2\Omega}$ using Thevenin's Theorem

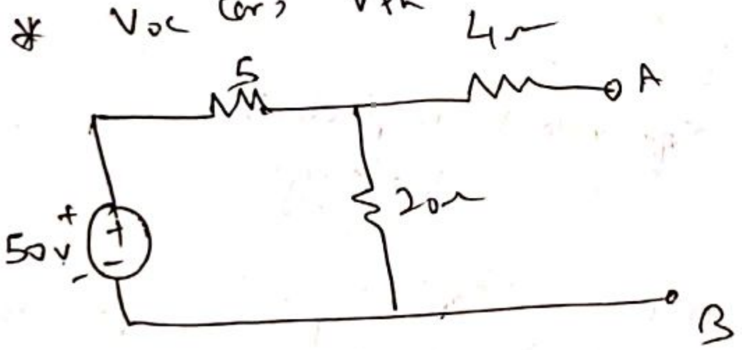



Sol/ * S.C. Vsrc & O.C. Isrc

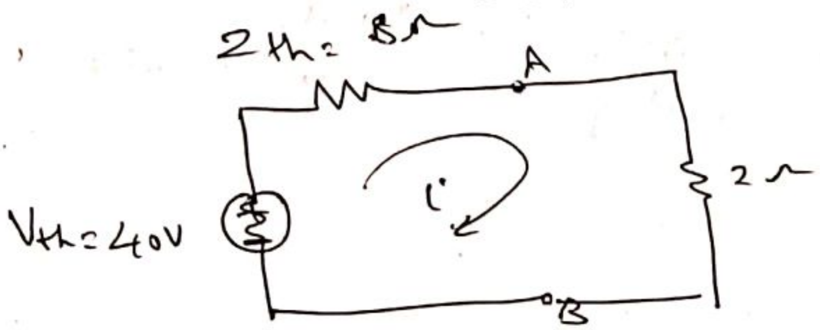



$Z_{th} = 8\Omega$

* $V_{oc}$ (or) $V_{th}$




$$V_{20\Omega} = V_{oc} = \frac{20 \times 50}{20+5} = 40V$$




$$i_{2\Omega} = \frac{40}{8+2} = 4A$$

To find $R_{th}$ :

3 types of ckts are encountered

(i) ckt with only independent srcs & resistors.

deactivate src & find $R_{th}$

[O.C. I src & S.C. V src]

(ii) ckt - Resistors, dep & Indepen src.

(a) Determine O.C. voltage $V_{oc}$ with the src activated

(b) Find the S.C. current $I_{sc}$, when S.C is applied to terminals a-b

(c) $R_t = \frac{V_{oc}}{i_{sc}}$.

(iii) If the ckt contains resistors & only dependent src

(a) $V_{oc} = 0$ (since there is no energy src)

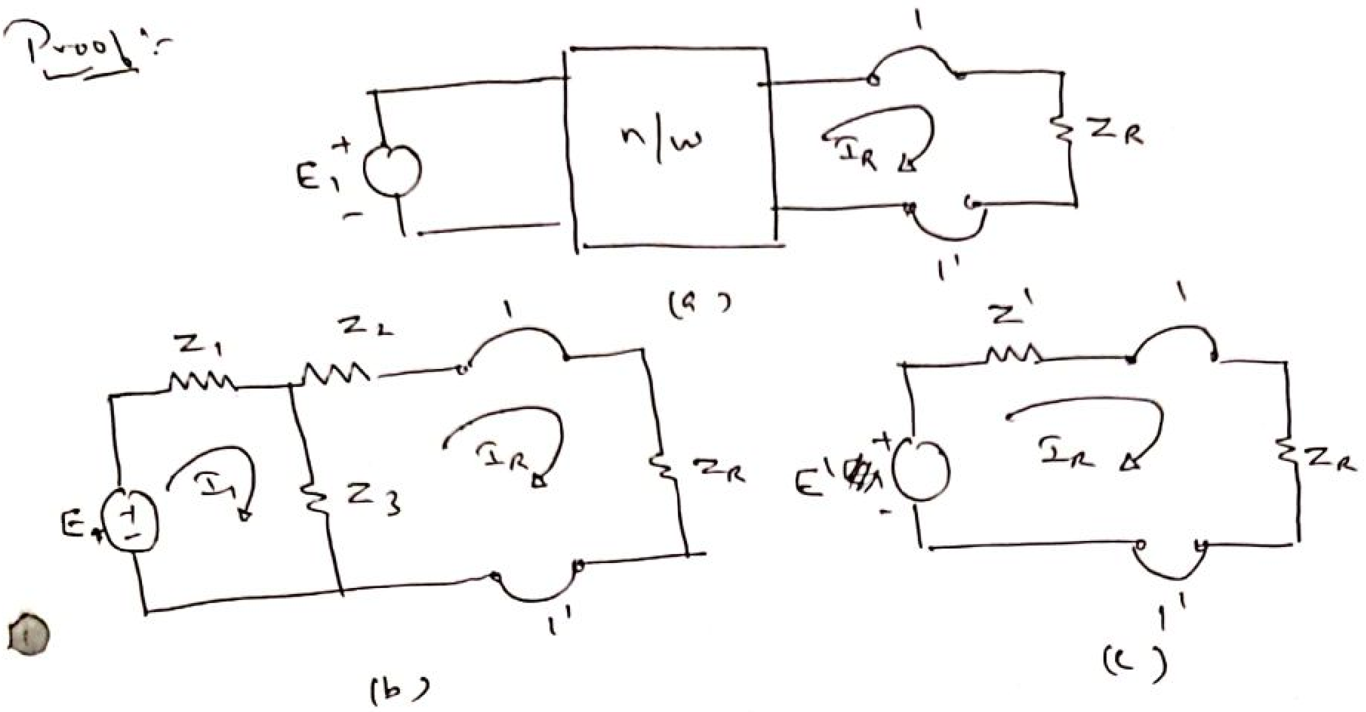
(b) Connect 1A current src to terminals a-b & determine $V_{ab}$

(c) $R_t = \frac{V_{ab}}{1}$.

Thevenin's Theorem



Proof:-




from fig (b)

$$E_1 = I_1 (Z_1 + Z_3) - I_R Z_3 \quad \text{——— (1)}$$

$$0 = - I_1 Z_3 + I_R (Z_2 + Z_3 + Z_R) \quad \text{——— (2)}$$

then $$I_1 = I_R \left( \frac{Z_2 + Z_3 + Z_R}{Z_3} \right) \quad \text{——— (3)}$$

$\therefore$ from eq (1)

$$I_R = \frac{E_1 \left( \frac{Z_3}{Z_1 + Z_3} \right)}{Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3} + Z_R} \quad \text{——— (4)}$$

from fig (b) voltage across o.c., 1 & 1'

is $E' = \frac{E_1 Z_3}{Z_1 + Z_3}$ & $Z' = Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3}$

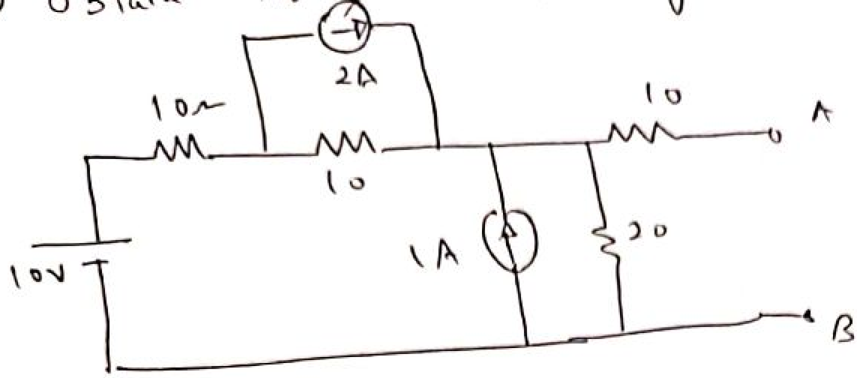
from fig (c)

$$I_R = \frac{E'}{Z' + Z_R} \quad \text{——— (5)}$$

eq (4) & (5) are same. Hence proof.

Problems:

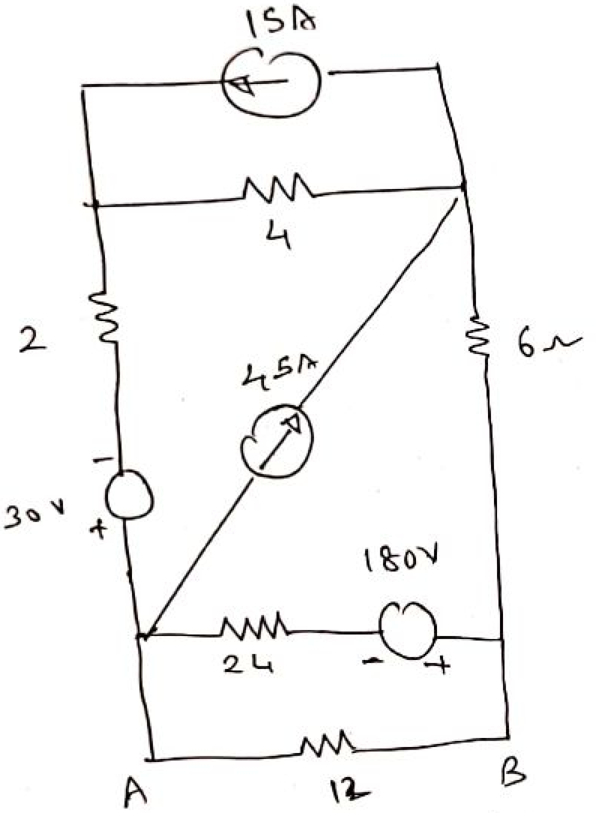
1) Obtain the Thevenin's Equivalent Ckt




$R_{th} = 20\Omega$

$V_{th} = 25V.$

2)

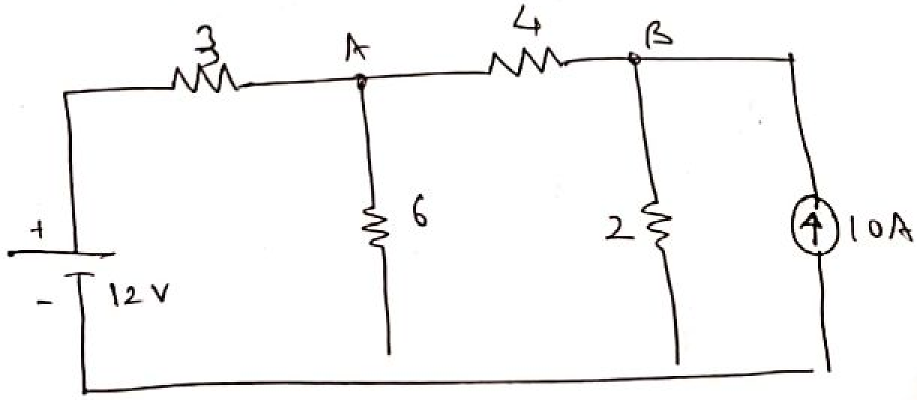



$V_{th} = -180V$

$R_{th} = 8\Omega$

$I_{12\Omega} = -9A$

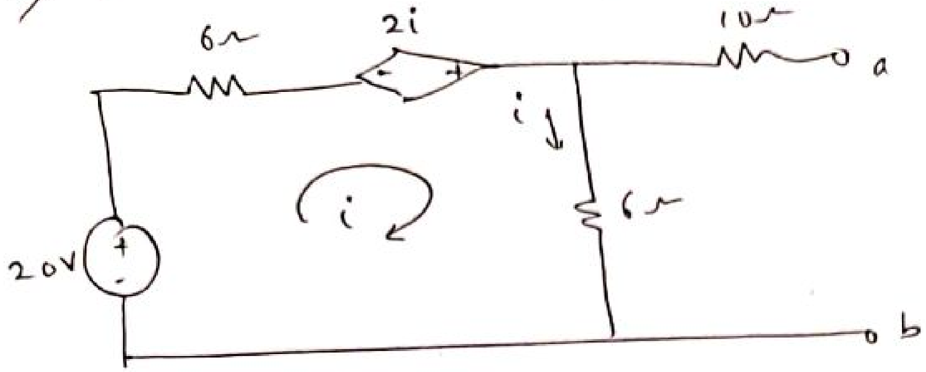
3)




$R_{th} = 4\Omega$

$V_{th} = -12V$

4) Find the Thevenin equivalent wrt a-b.



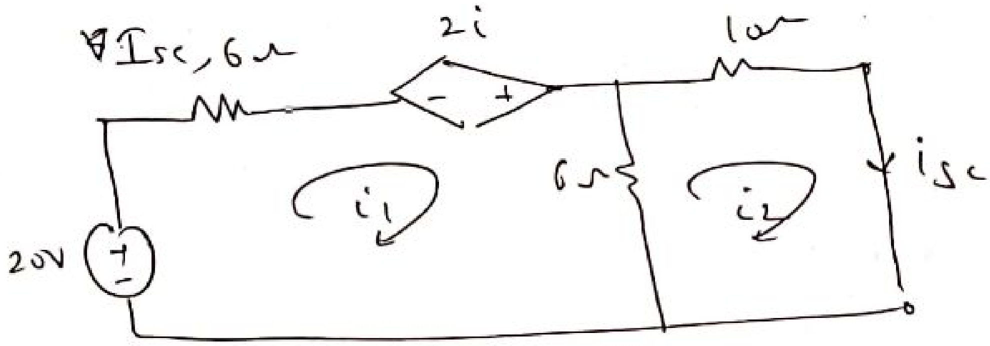

Sol) $V_{oc}$,

$$+20 - 6i + 2i - 6i = 0$$

$$i = 2\,A$$

$$\therefore V_{oc} = 6 \times 2 = 12\,V$$

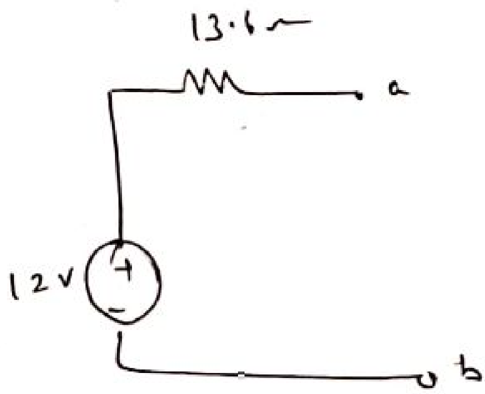
$I_{sc}$,



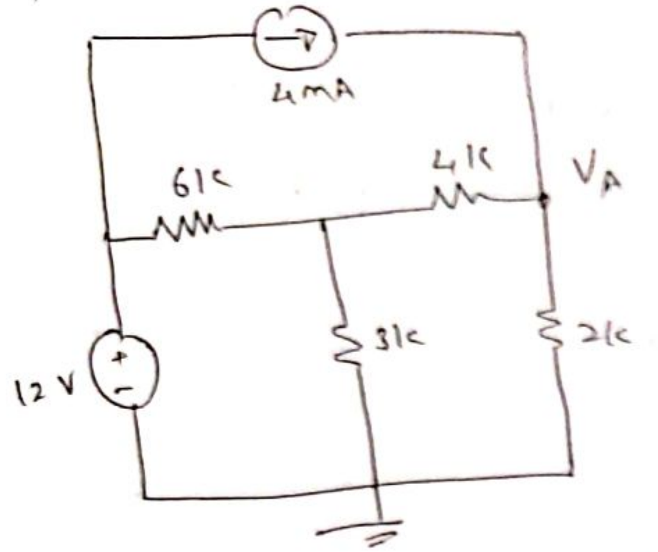

$$i_2 = 0.88\,A.$$

$$i_{sc} = i_2 = 0.88\,A$$

$$\therefore R_t = \frac{V_{oc}}{i_{sc}} = 13.6\,\Omega$$

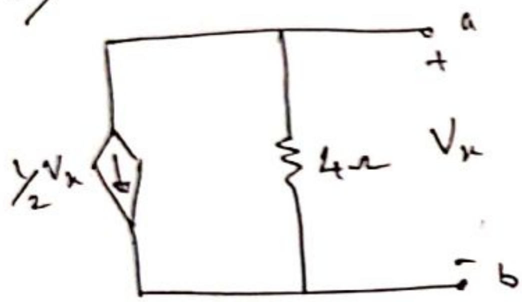


5) Find the $V_A$ using thevenin's theorem

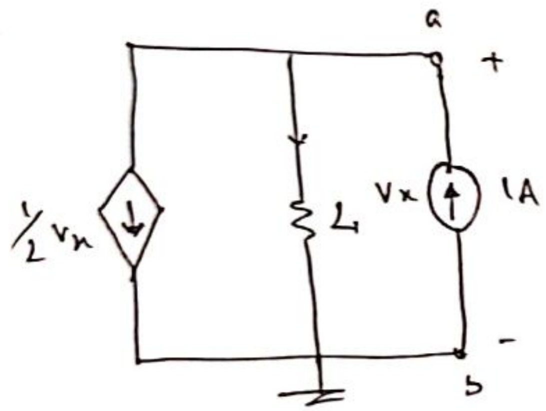



$V_A = 7V.$

6) Thevenin's b/w A & B



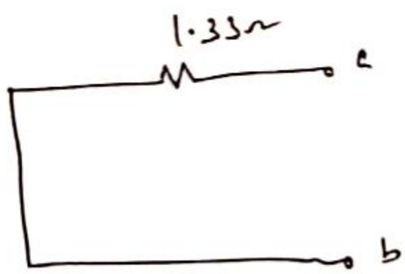

s)




$$\frac{1}{2}V_x + \frac{V_x}{4} = 1$$

$$V_x = 1.33V.$$

$$R_t = \frac{V_x}{1} = 1.33\Omega$$



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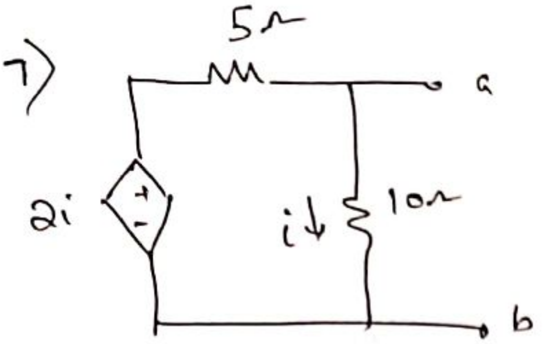
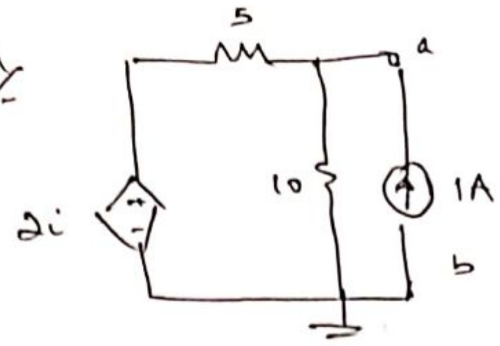



Sol/-

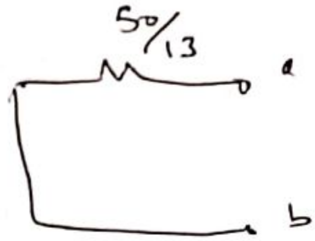



Sol/-

$$\frac{V_a - 2i}{5} + \frac{V_a}{10} = 1$$

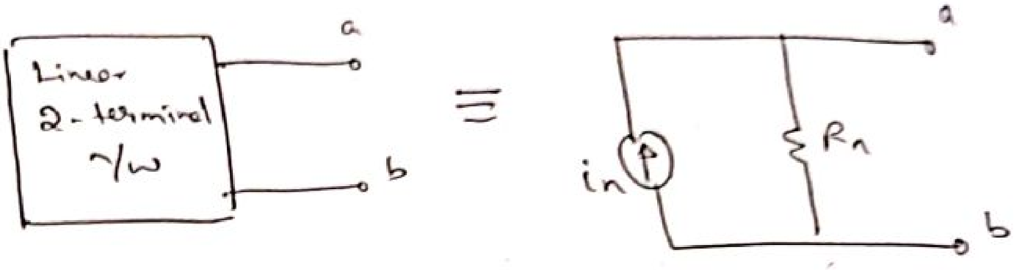
$$V_a = \frac{50}{13}\,V$$

$$R_t = \frac{V_a}{1} = \frac{50}{13}\,\Omega$$



## Norton's Theorem :-

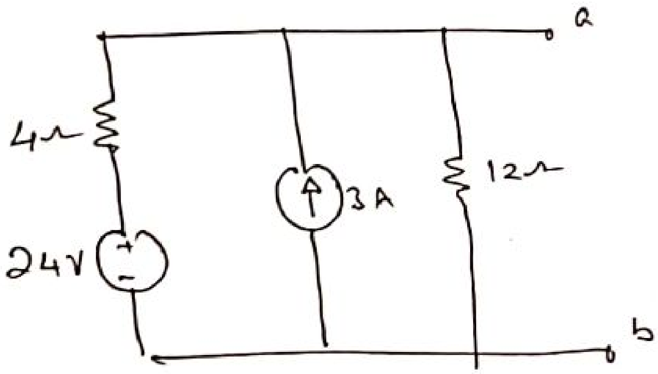
" A linear 2-terminal n/w can be replaced by an equivalent ckt consisting of a current src $i_n$ in $\parallel^{le}$ with resistor $R_n$. $i_n$ is the s.c. current through the terminals & $R_n$ is the equivalent resistance at the terminals when the independent src are deactivated."



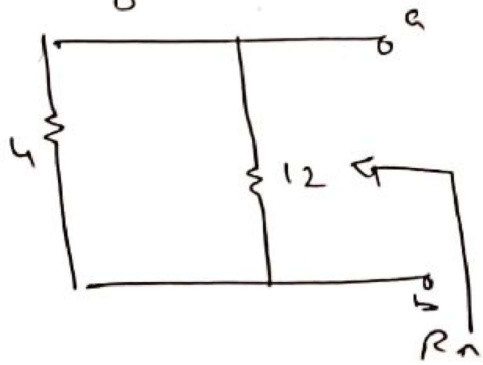

$R_n = R_{th}$, & $i_n = \frac{V_{oc}}{R_{th}}$.

## Problems:

1) Find the Norton Equivalent

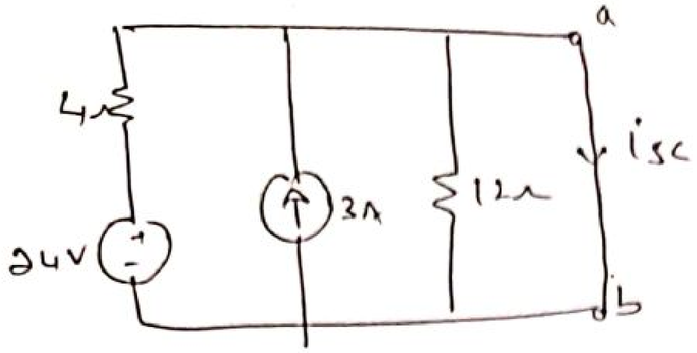


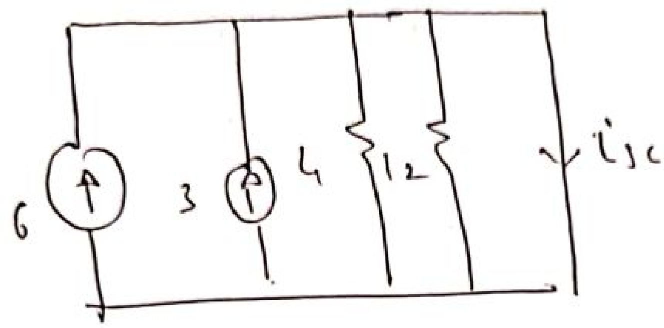

Sol:- To find $R_n$



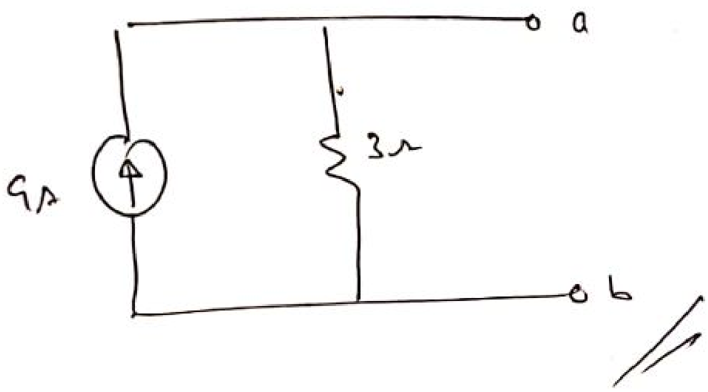

$\therefore R_n = 3\Omega$

To find $i_n$ (or) $i_{sc}$



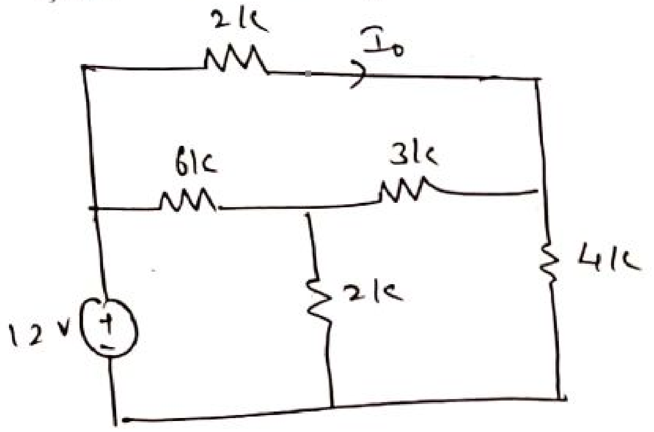




$\therefore \quad i_{sc} = 9A$



33 a

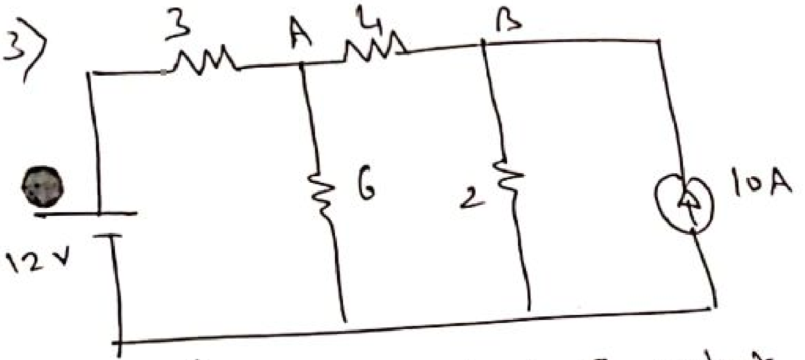
2) find $I_o$ using Norton's theorem




$I_o = 2.57\,mA$

$[R_n = 2.12k, \ I'_{sc} = 5mA]$

3)

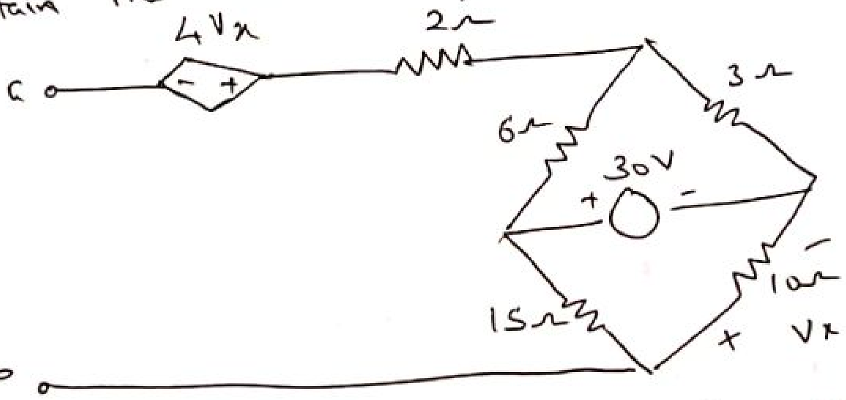



$I_L = 1.5\,A.$

$I_{SC} = -3\,A.$

$R_n = 4\,\Omega$

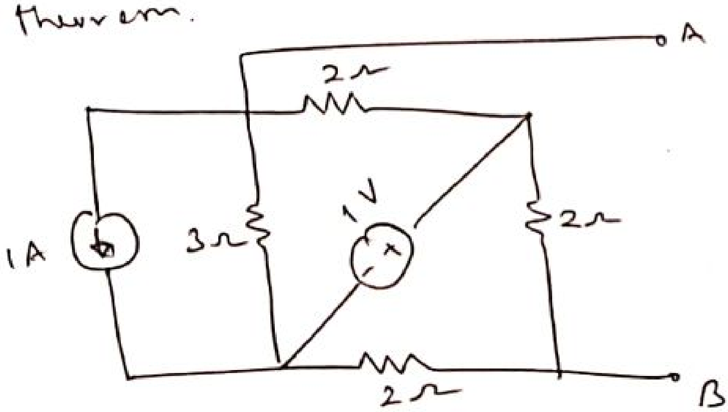
4) Obtain the Norton's Equivalent of the n/w shown




$R_{th} = 34\,\Omega$

$I_n = 1.4706\,A$

5) Determine the Current through 1Ω resistor Connected across AB in the n/w shown using Norton's theorem.




$I_{1\Omega} = 0.344\,A$

$R_n = 2.2\,\Omega, \ I_n = 0.5\,A$

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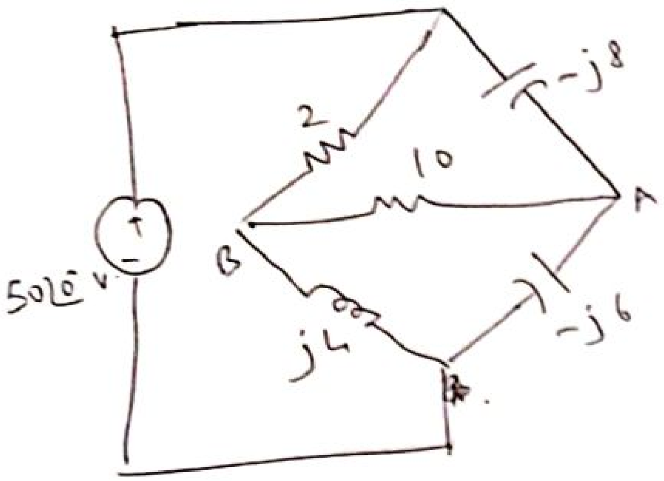
6) Find the Thevenin's Equivalent ckt for the portion of the n/w external to the elements b/w A & B



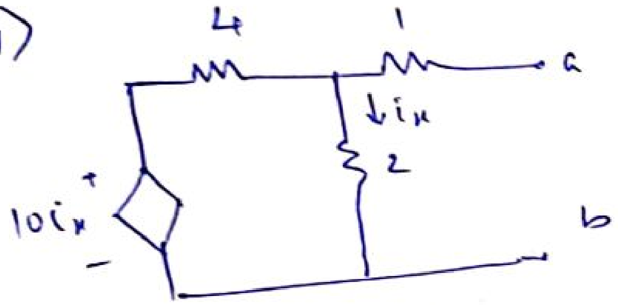

$R_{th} = 6.83 \angle -54.17° \Omega$

$V_{th} = 30.60 \angle 38.83° V$

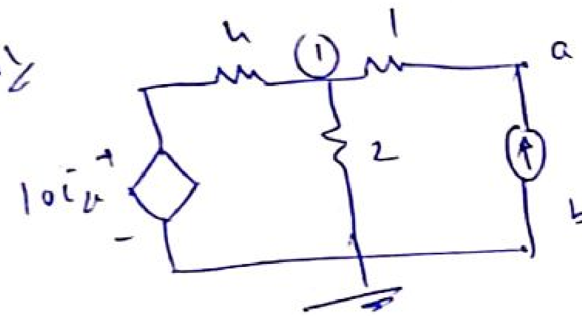
$R_{th} = 3.01 \angle -56.50° \Omega$

$V_{th} = 27.2 \angle -47.1 V$

7)




Sol




$i_x = \frac{V_1}{2}$

$$\frac{V_1 - 10 i_x}{4} + \frac{V_1}{2} = 1$$

$$\frac{V_1 - 5V_1}{4} + \frac{V_1}{2} = 1$$

$V_1 = -2$

$$\frac{V_{oc} - V_1}{1} = 1$$

$V_{oc} = 1 + V_1$

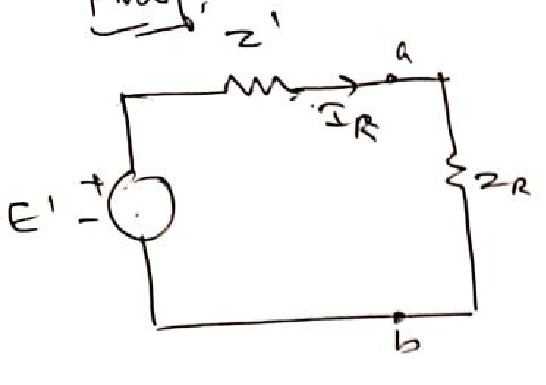
$V_{oc} = -1$

$R_{th} = \frac{V_{oc}}{1} =$

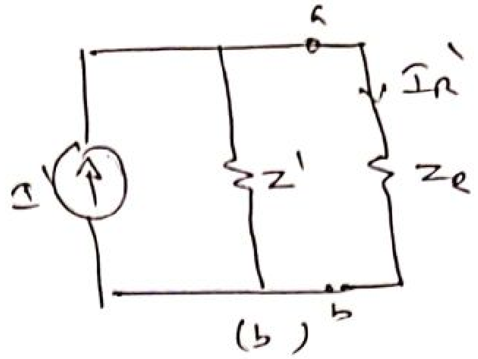
$R_{th} = -1$

## Norton's Theorem

**Proof:**




(a)




(b)

from fig (a)

$$I_R = \frac{E'}{Z'+Z_R} = \left(\frac{E'}{Z'}\right)\left(\frac{Z'}{Z'+Z_R}\right)$$

$$\cancel{I_R = I'\frac{Z'}{Z'+Z_R} = \frac{E'}{\frac{1}{y'}+\frac{1}{y_R}}}$$

$$I_R = \frac{E'}{Z'}\left(\frac{y_R}{y'+y_R}\right) = \cancel{I'\left(\frac{y_R}{y'+y_R}\right)} \quad \text{--- (1)}$$

when $z' = 1/y'$ & $z_R = 1/y_R$.

fig (b)

$$I_R' = I'\left(\frac{Z'}{Z'+Z_R}\right)$$

$$I_R' = I'\left(\frac{y_R}{y'+y_R}\right) \quad \text{--- (2)}$$

eq (1) & (2) are same if $I' = \frac{E'}{Z'} = E'y'$

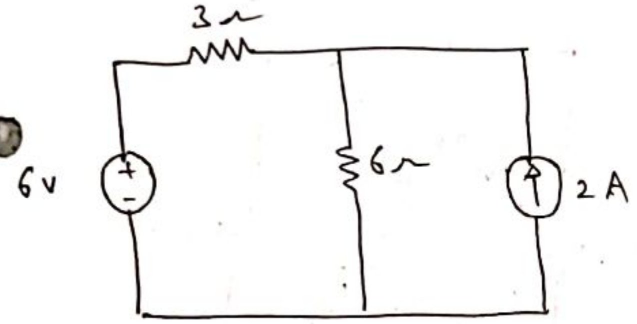
when $y' = \frac{1}{Z'}$ //

## Superposition Theorem:

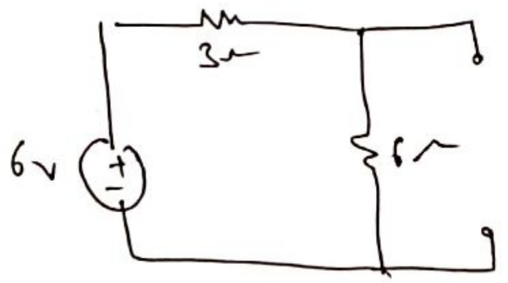
"In any linear ckt containing multiple independent srces, the current (or) voltage at any point in the n/w may be calculated as algebraic sum of the individual contribution of each source acting alone.

### Problems:

1) Find $i_{6\Omega}$ using Principle of Superposition.

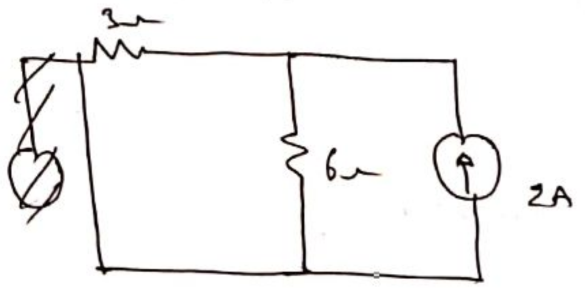



Sol:- Set $I_{src}$ to zero




$$i'_{6\Omega} = \frac{6}{9} = 0.666A$$

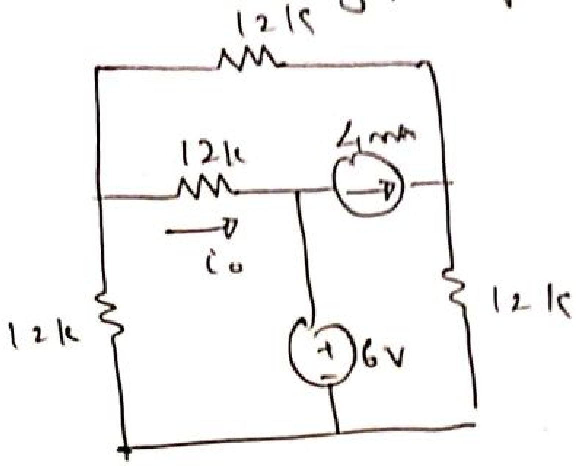
Set $V_{src}$ to zero



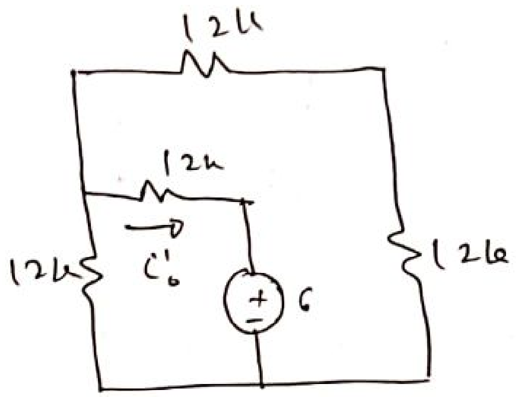

$$i''_{6\Omega} = \frac{2 \times 3}{9} = 0.666A$$

$\therefore$ $i_{6\Omega} = 1.33A$

2) Find $i_o$ using principle of Superposition

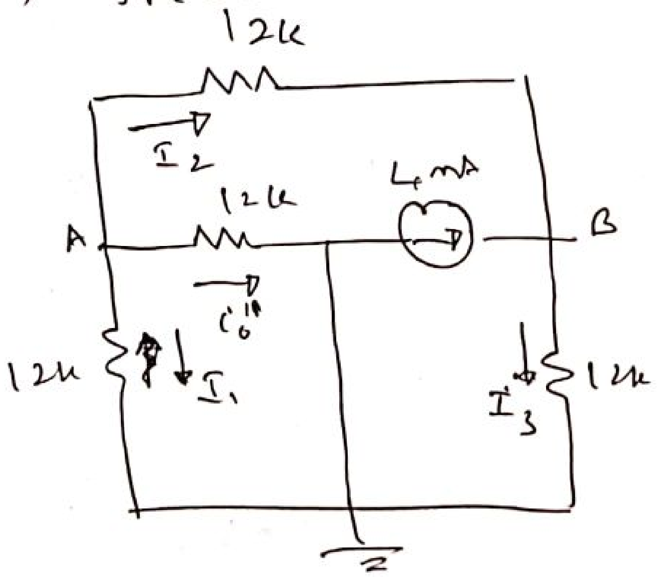



S-> a) $I_{src} = 0$




$$i_o'' = \frac{-6}{(8+12)k} = -0.3\,mA$$

b) $V_{src} = 0$




$\cancel{i_o'' = 4mA}$

$\cancel{i_o = i_o' + i_o'' = 4.3\,mA}$

@ A,
$$\frac{V_A}{12k} + \frac{V_A - V_B}{12k} + \frac{V_A}{12k} = 0$$

$$V_A\left[\frac{3}{12k}\right] - \frac{V_B}{12k} = 0 \quad \text{———} (1)$$

@ B,

$$4\times10^{-3} + \frac{V_A - V_B}{12k} = \frac{V_B}{12k}$$

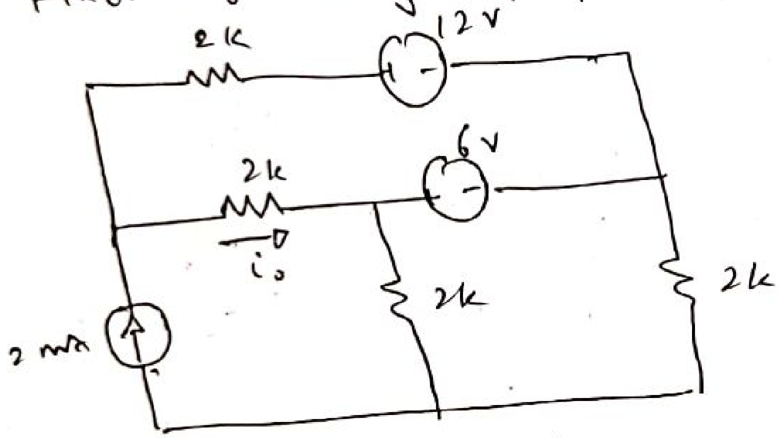
$$-\frac{V_A}{12k} + V_B\left[\frac{2}{12k}\right] = 4\times10^{-3} \quad \text{———} (2)$$

$V_A = 9.6V$, $V_B = 28.8V$

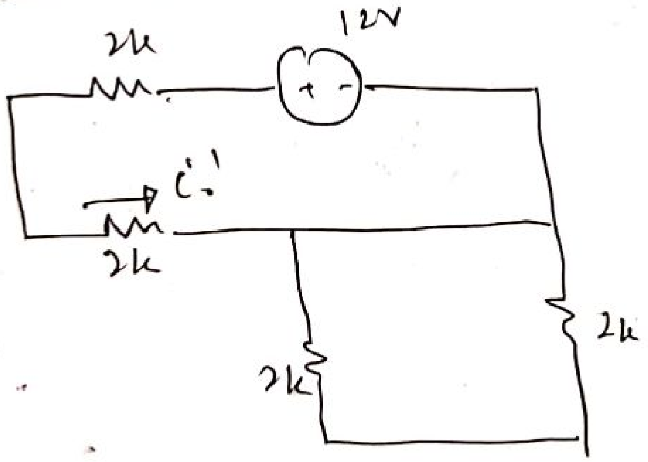
$$\therefore i_o'' = \frac{9.6}{12k} = 0.8mA$$

$$i_o = i_o' + i_o'' = 0.5mA$$

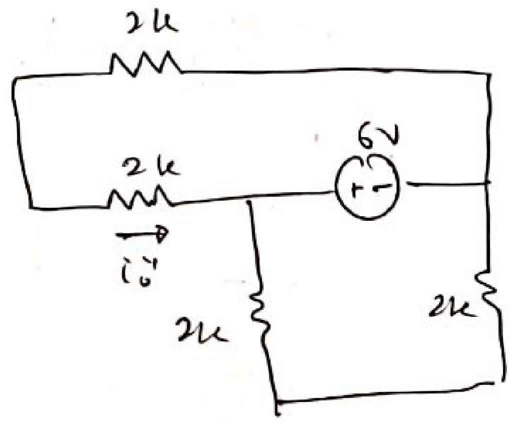
3) Find $i_o$ using Superposition.




Sol a)



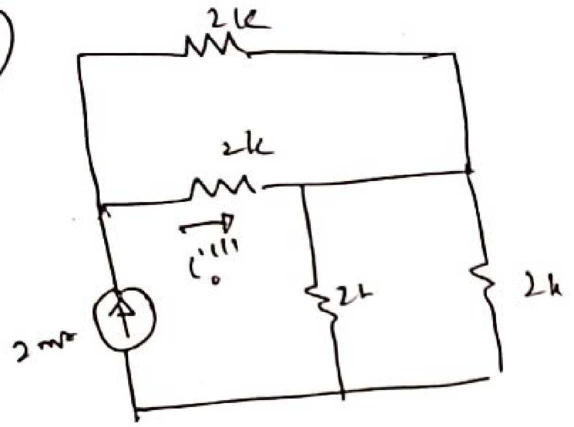

b)

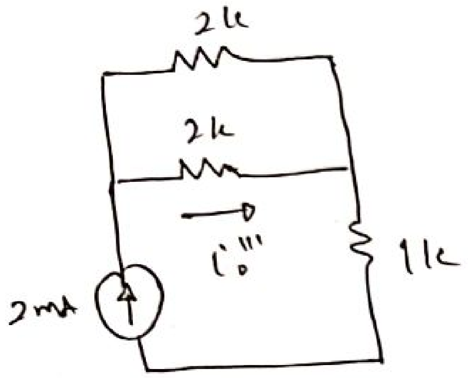



$i_o' = 3mA$

$i_o'' = -1.5mA$

c)

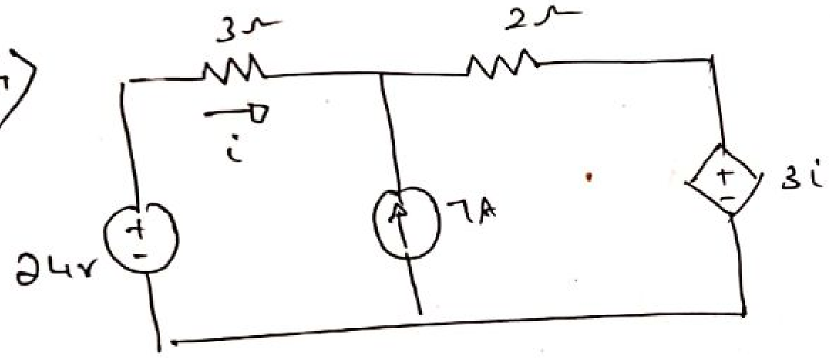


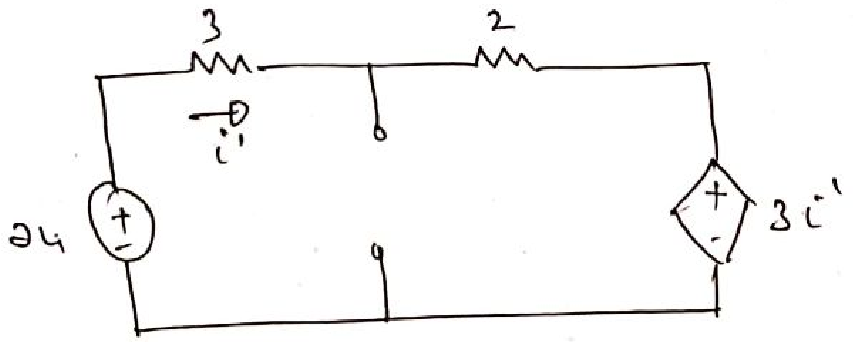

$$i_o''' = \frac{2mA \times 2k}{4k} = 1mA$$

$$i_o = i_o' + i_o'' + i_o''' = 2.5mA$$

4)

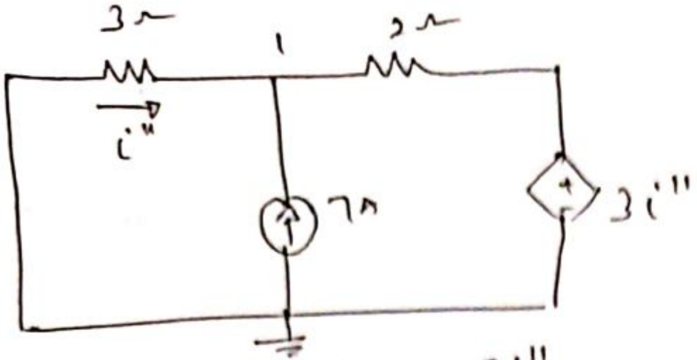



find i




$$i' = \frac{24 - 3i'}{5}$$

$$i' = 3A$$




$$i'' + 7 = \frac{V_1 - 3i''}{2}$$

$$i'' = \frac{-V_1}{3}$$

$$i'' + 7 = \frac{-3i'' - 3i''}{2}$$

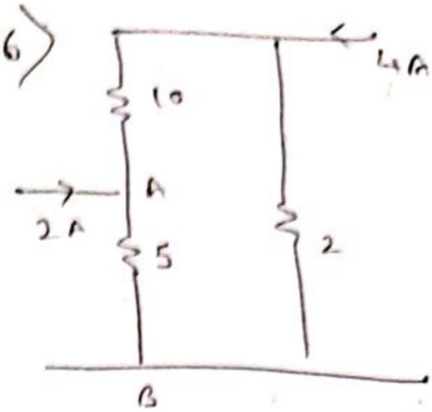
$$2i'' + 14 = -6i''$$

$$8i'' = -14$$

$$i'' = -7/4$$

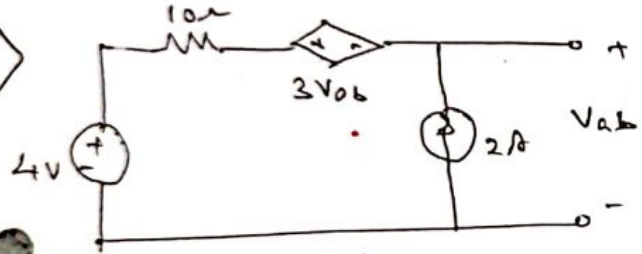
$\therefore\ i = i' + i'' = 3 - 7/4 = 5/4\ A$

6)



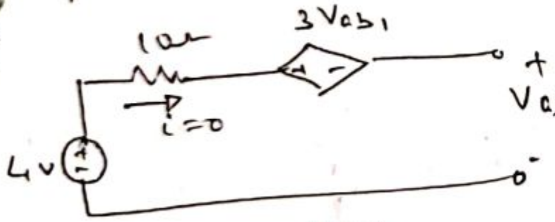

Ans $V_{ab} = 9.41V$

5)



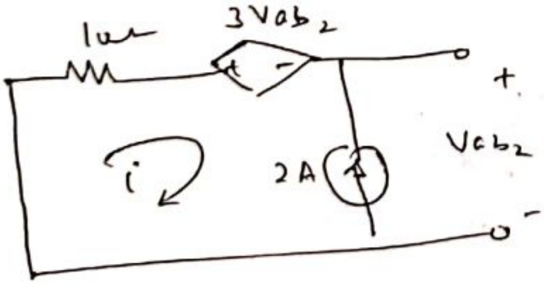

Apply Superposition & find Vab.

Sol




$$4 - 3V_{ab_1} - V_{ab_1} = 0$$

$$V_{ab_1} = 1V$$




$$+10i - 3V_{ab_2} - V_{ab_2} = 0$$

$$-20 - 4V_{ab_2} = 0$$

$$V_{ab_2} = 5V$$

$\therefore\ V_{ab} = V_{ab_1} + V_{ab_2} = 6V$

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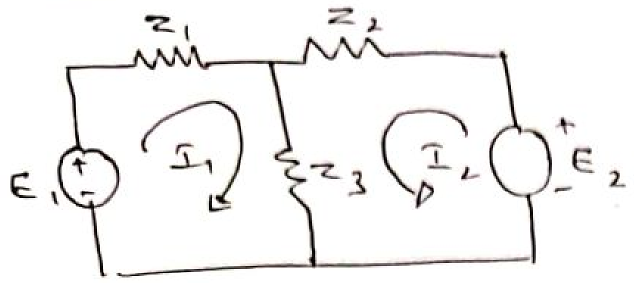
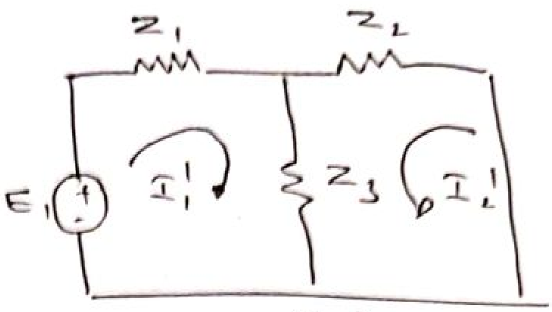
## Superposition theorem:

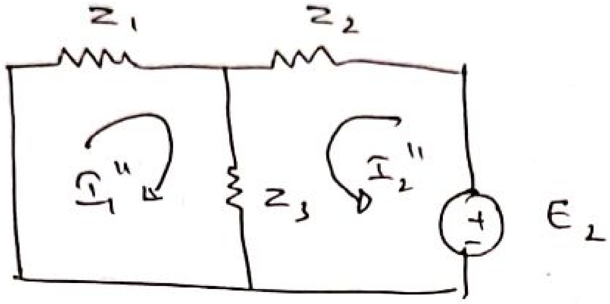
**Proof:-** Consider linear n/w shown below,




(a)




(b)




(c)

The Current flowing in any element is the vector sum of the Currents that are sepeeately Caused to flow in that element by each energy src.

Consider fig(a)

Apply KVL to loop ① & ② we have

$$E_1 = I_1(Z_1+Z_2) + I_2 Z_3 \quad \&$$

$$E_2 = I_1(Z_3) + I_2(Z_2+Z_3) \text{ respectively}$$

Solving above eq's

$$I_1 = \left(\frac{Z_2+Z_3}{\Sigma Z}\right)E_1 - \left(\frac{Z_3}{\Sigma Z}\right)E_2 \quad \text{---} (1)$$

$$I_2 = \left(\frac{-Z_3}{\Sigma Z}\right)E_1 + \left(\frac{Z_1+Z_3}{\Sigma Z}\right)E_2 \quad \text{---} (2)$$

When $\Sigma Z = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$ //

Making $E_2 = 0$, fig (3)

$$E_1 = I_1'(Z_1 + Z_2) + I_2' Z_3$$

$$0 = I_1'(Z_3) + I_2'(Z_2 + Z_3)$$

Solving above (2) equations

$$I_1' = \left(\frac{Z_2 + Z_3}{\Sigma Z}\right) E_1 \quad \text{——— (3)}$$

$$I_2' = \left(\frac{-Z_3}{\Sigma Z}\right) E_1 \quad \text{——— (4)}$$

Making $E_1 = 0$, fig (1)

$$0 = I_1''(Z_1 + Z_2) + I_2'' Z_3$$

$$E_2 = I_1''(Z_3) + I_2''(Z_2 + Z_3)$$

Solving above 2 eqn

$$I_1'' = \left(\frac{-Z_3}{\Sigma Z}\right) E_2 \quad \text{——— (5)}$$

$$I_2'' = \left(\frac{Z_1 + Z_3}{\Sigma Z}\right) E_2 \quad \text{——— (6)}$$

eq (3) + (5), we have

$$I_1' + I_1'' = \left(\frac{Z_2 + Z_3}{\Sigma Z}\right) E_1 + \left(\frac{-Z_3}{\Sigma Z}\right) E_2$$

from eq (1) we have

$$\boxed{I_1 = I_1' + I_1''}$$

|||ly from eq (4), (6) & (2)

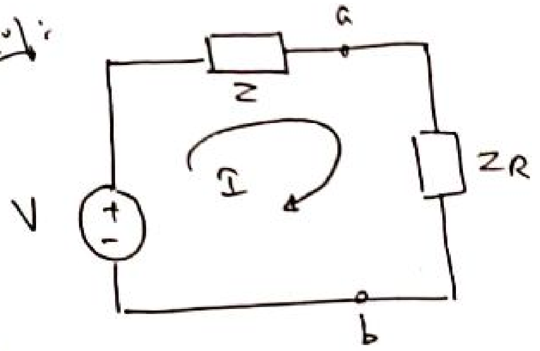
$$\boxed{I_2 = I_2' + I_2''}$$

Hence SPT is proved.

# Maximum Power Transfer Theorem.

**General:** " Max. power will be delievered by a n/w, to an impedance $Z_R$ if the impedance $Z_R$ is the complex conjugate of the impedance of the n/w measured looking back into the terminals of the n/w".

**Proof:**




$$I = \frac{V}{Z + Z_R}$$

$$= \frac{V}{(R_R + R) + j(X_R + X)}$$

But Power ~~destroyed~~ delievered to the load is

$$|I| = \frac{V}{\sqrt{(R_R+R)^2 + (X_R+X)^2}} \quad \text{——} (1)$$

$$P = I^2 R_R$$

$$P \cancel{= \frac{V^2}{[(R_R+R)+j(X_R+X)]^2}} = \cancel{\frac{V^2}{\quad}}$$

$$\therefore P = \frac{V^2 R_R}{(R_R+R)^2 + (X_R+X)^2} \quad \text{——} (2)$$

For Max. power. $\frac{\partial P}{\partial X_R}$ must be zero

$$\frac{\partial P}{\partial X_R} = \frac{0 - 2(V^2) R_R (X_R + X)}{\left[[R_R+R]^2 + [X_R+X]^2\right]^2} = 0$$

$$X_R + X = 0$$

$$\boxed{X_R = -X}$$

i.e the reactance of the load impedance is of opposite sign to the reactance of the src impedance

Substitute, $X_R = -X$ in eq (2)

$$P = \frac{V^2 R_R}{(R_R+R)^2 + (X_R - X_R)^2}$$

$$P = \frac{V^2 R_R}{(R_R+R)^2}$$

for Max. power

$$\frac{\partial P}{\partial R_R} = 0$$

$$\frac{\partial P}{\partial R_R} = \frac{V^2 (R_R+R)^2 - 2(E)^2 R_R (R_R+R)}{[R_R+R]^4} = 0$$

$$V^2 \left[ (R_R+R)^2 - 2R_R (R_R+R) \right] = 0$$

$$V^2 (R_R+R) \left[ R_R + R - 2R_R \right] = 0$$

$$\boxed{\therefore R_R = R}$$

$\therefore$ $X_R = -X$ & $R_R = R$, the Max. power will be transfered from the src to load

i.e for Max. power transfer the load impedance $Z_R$ should be complex Conjugate of internal impedance of the src i.e $\boxed{Z_R = Z^*}$

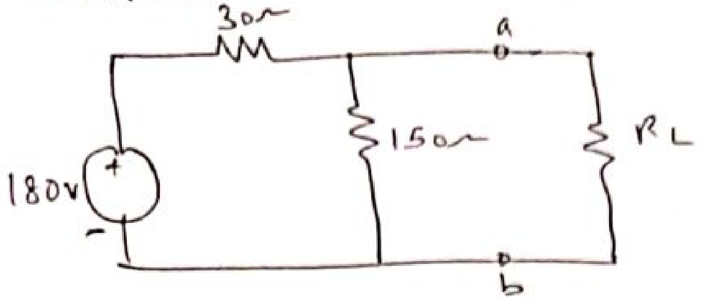
$\therefore$ The Max. power transfered will be

$$P_m = \frac{V^2 R_R}{(R_R + R_R)^2 + (X_R - X_R)^2}$$

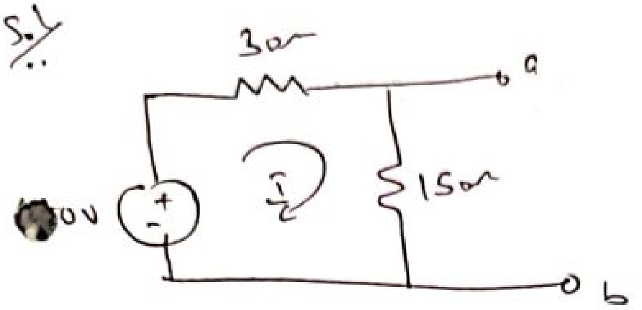
$$\therefore \boxed{P_m = \frac{V^2}{4 R_R}}$$

Problems:

1) Find the load $R_L$ that will result in max. power delivered to the load for the ckt. & also find $P_{max}$




Sol:

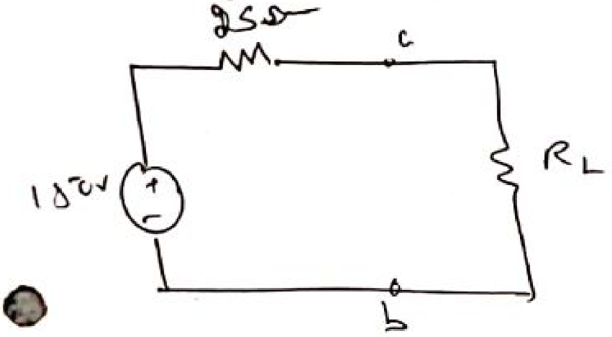



$R_{th} = 25Ω$

$V_{th} = I \times 150 = 150V$

$I = 1A$

Rewriting the ckt,

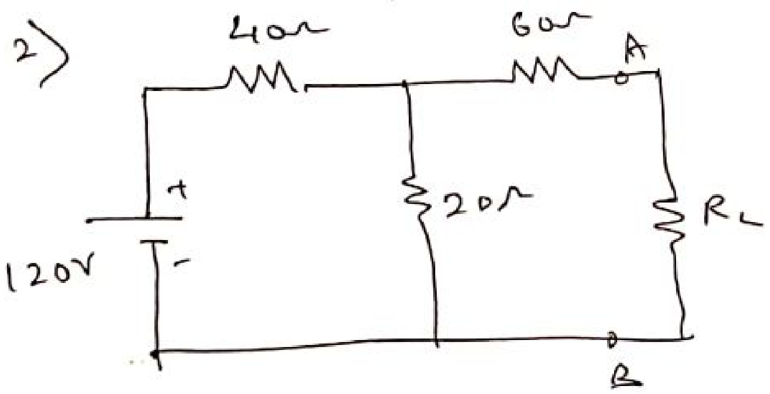



W.K.T

$\therefore R_L = R_{th} = 25Ω$

$\therefore P = \frac{V^2}{4R_L} = 225W$

2)




$R_L = 73.33Ω$

$P_{max} = 5.45W$

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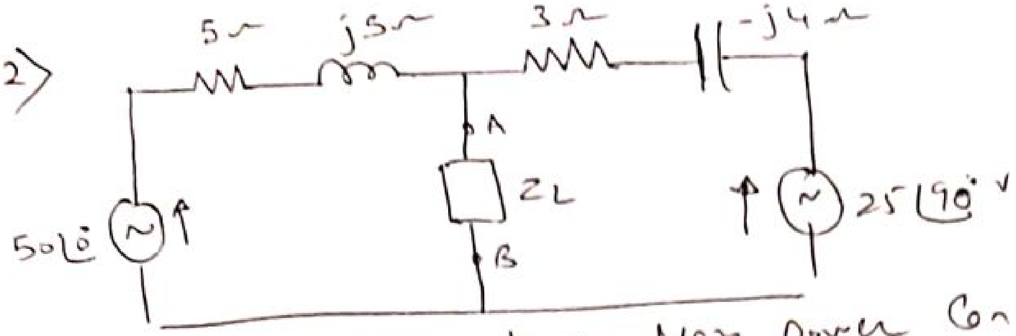
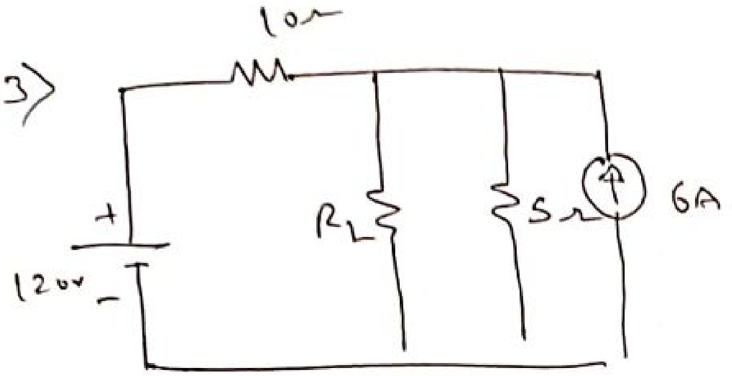
2)




Find $Z_L$ such that Max power can be received by it. (Ans: $Z_L = (4.23 + j1.154)\,\Omega$
$P = 50.822$

3)




$R_L = 3.33\,\Omega$

$P_{max} = 270W.$

MPT :-

" In any Linear bilateral n/w, Max. power is transfered from Src to the load when

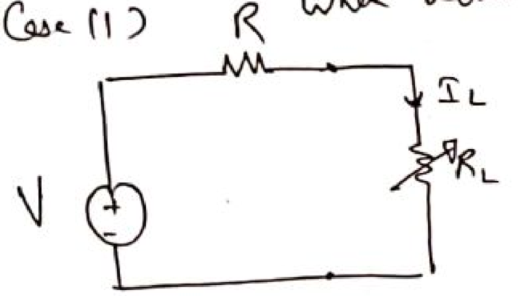
Case (1) The Load Resistance is equal to the Src resistance

Case (2) The Load Resistance is equal to the magnitude of the Src impedance

Case (3) The load impedance is Complex Conjugate of the Src impedance

Sol

Case (1) When Load & Src are purely resistive




from the ckt,

$$I_L = \frac{V}{R+R_L} \quad \text{——} \quad (1)$$

But

$$P = I_L^2 R_L = \frac{V^2 R_L}{(R+R_L)^2}$$

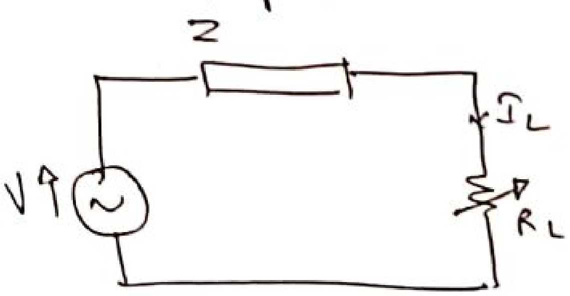
The power transfered is Max, when $\frac{\partial P}{\partial R_L} = 0$

$$\frac{(R+R_L)^2 V^2 - V^2 R_L \, 2(R+R_L)}{[R+R_L]^4} = 0$$

$$\boxed{R_L = R}$$

$$\boxed{\therefore P_m = I_L^2 R_L = \frac{V^2}{4R_L}}$$

Case (2) When load is purely resistive & the Src has impedance




Let Z be the internal impedance of the Src

$$I_L = \frac{V}{(R+R_L)^2 + jX}$$

Where $Z = R + jX$

$$|I_L| = \frac{V}{\sqrt{(R+R_L)^2 + X^2}}$$

$\therefore$ Power is,

$$P = I_L^2 R_L = \left[\frac{V^2}{\left[\sqrt{(R+R_L)^2+X^2}\right]^2}\right] R_L$$

$$P = \frac{V^2 R_L}{(R+R_L)^2 + X^2}$$

$$\frac{\partial P}{\partial R_L} = 0 \qquad \therefore \quad R_L^2 = R^2 + X^2$$

ie $\boxed{R_L = \sqrt{R^2 + X^2}} = |Z|$

Max. power is transferred when $R_L = |Z|$

$\boxed{P_m = I_L^2 R_L}$ when $R_L = |Z|$

$P_m =$

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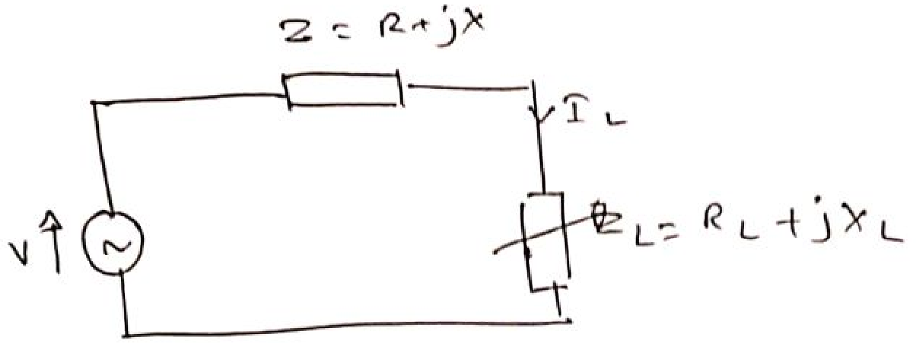
Case (3) When both load & Src have impedance




wkt.
$$I_L = \frac{V}{(R+R_L)+j(X+X_L)}$$

$$|I_L| = \frac{V}{\sqrt{(R+R_L)^2+(X+X_L)^2}}$$

But $P = I_L^2 R_L$

$$P = \frac{V^2 R_L}{(R+R_L)^2+(X+X_L)^2}$$

The power transfered is Max when $\boxed{X_L = -X_0}$ —(1)

$$\therefore P_1 = \frac{V^2 R_L}{(R+R_L)^2}$$

Power is Max if $\frac{\partial P_1}{\partial R_L} = 0$

$$\boxed{R_L = R} \quad \text{—(2)}$$

$\therefore$ from (1) & (2) $R_L + jX_L = R + jX$

$$\boxed{Z_L = Z^*}$$

$$\boxed{P_m = \frac{V^2}{4R_L}}$$

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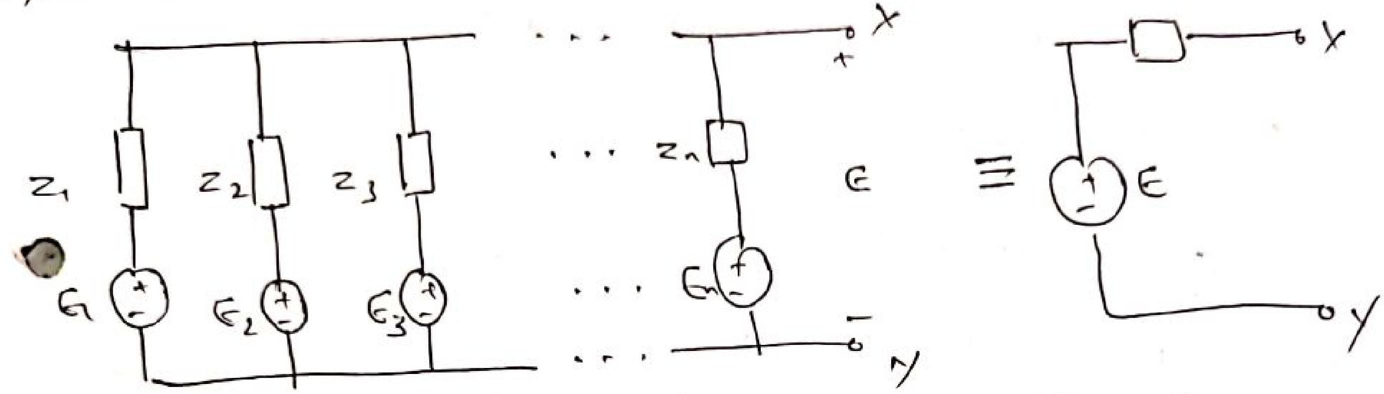
Millman's Theorem :-

" If there are a number of voltage srces in ll^l operating conjunction, they may be replaced by a single equivalent voltage src " connected in ll^l

Proof: Let $E_1, E_2, E_3, \ldots E_n$ be the voltage srces with internal impedances $Z_1, Z_2, Z_3 \ldots Z_n$ respectively, b/w x-y connected as shown in the fig.

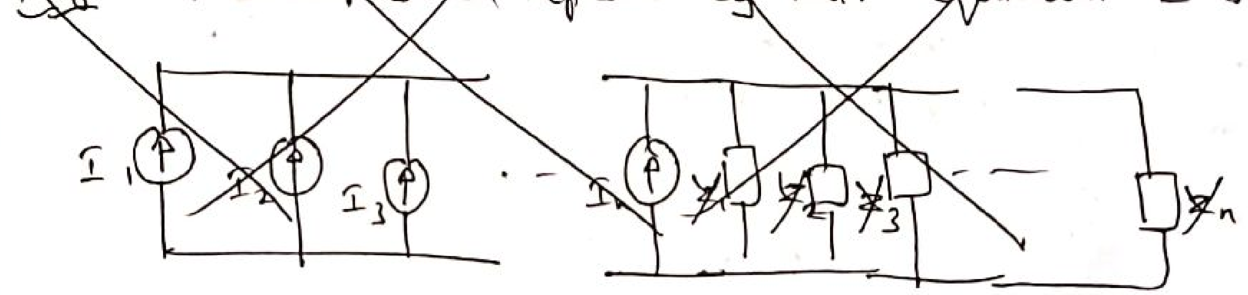


" It states that if there are n voltage src (b/w x & y) can be replaced by a single equivalent src with voltage $E'$ in series with an equivalent impedance $z'$ as given by,

$$E' = \frac{E_1 Y_1 + E_2 Y_2 + \ldots + E_n Y_n}{Y_1 + Y_2 + \ldots + Y_n}$$

$$\& \; z' = \frac{1}{Y} = \frac{1}{Y_1 + Y_2 + \ldots + Y_n}$$

Proof: V. srces can be replaced by their equivalent I. srces



Proof: Apply nodal analysis at x

$$\frac{E_1 - E}{Z_1} + \frac{E_2 - E}{Z_2} + \cdots + \frac{E_n - E}{Z_n} = 0$$

$$\left[\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \cdots + \frac{E_n}{Z_n}\right] = E\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}\right]$$

$$E_1 Y_1 + E_2 Y_2 + \cdots + E_n Y_n = E\left[\frac{1}{Z}\right]$$

where $Z$ = Equivalent internal impedance

i.e $E_1 Y_1 + E_2 Y_2 + \cdots + E_n Y_n = E Y$

$$\boxed{E = \frac{E_1 Y_1 + E_2 Y_2 + \cdots + E_n Y_n}{Y}}$$

where $Y = Y_1 + Y_2 + \cdots + Y_n$

$$\& \quad \boxed{Z = \frac{1}{Y} = \frac{1}{Y_1 + Y_2 + \cdots + Y_n}}$$

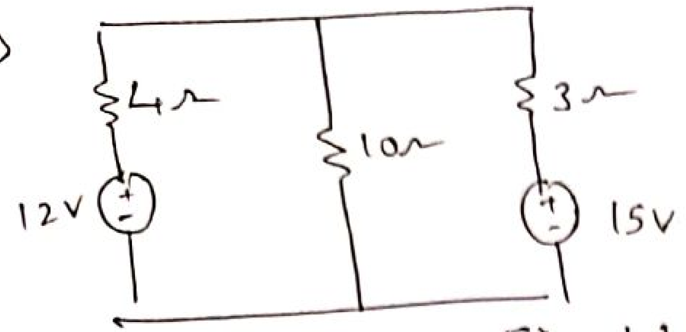
[OR]

"Millman's theorem states that if n number of generators having emfs $E_1, E_2, \cdots E_n$ & internal impedances $Z_1, Z_2 \cdots Z_n$ respectively are connected in $\parallel^{l}$, then the emfs & impedances can be combined to give a single equivalent emf of $E$ with an internal impedance of equivalent value $Z$.

$$\text{where } E = \frac{E_1 Y_1 + E_2 Y_2 + \cdots + E_n Y_n}{Y_1 + Y_2 + \cdots + Y_n}$$

$$\& \quad Z = \frac{1}{Y_1 + Y_2 + \cdots + Y_n}$$

Problems:

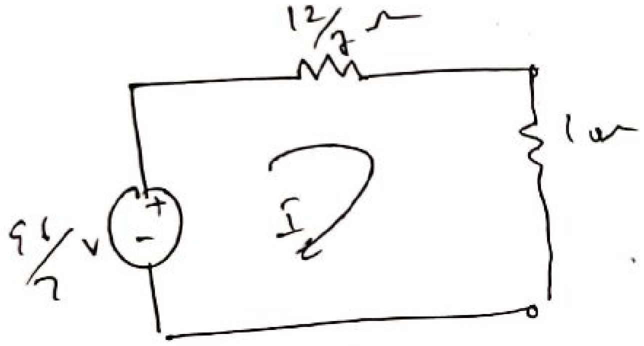
1) 


find $I_{10\Omega}$ using Millman's theorem.

$Z_1 = 4\Omega, Z_2 = 3\Omega \quad E_1 = 12V, \; \& \; E_2 = 15V$

$$E = \frac{E_1 Y_1 + E_2 Y_2}{Y_1 + Y_2} = \frac{12(1/4) + 15(1/3)}{1/4 + 1/3} = \frac{96}{7} V$$

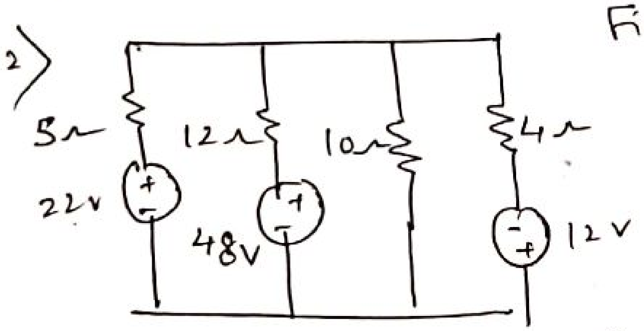
$$Z = \frac{1}{Y_1 + Y_2} = \frac{1}{1/4 + 1/3} = \frac{12}{7} \Omega$$




$$\therefore I_{10\Omega} = \frac{96/7}{12/7 + 10}$$

$$= 1.17 A$$

2) Find $I_{10\Omega}$



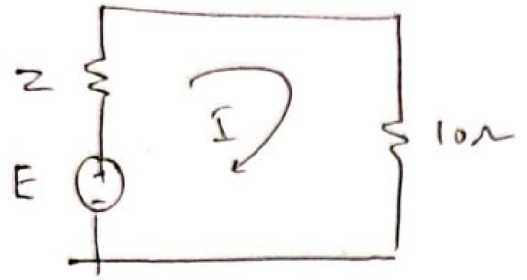

$E_1 = 22V, \quad E_2 = 48V$

$E_3 = -12V.$

$$E = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3}{Y_1 + Y_2 + Y_3}$$

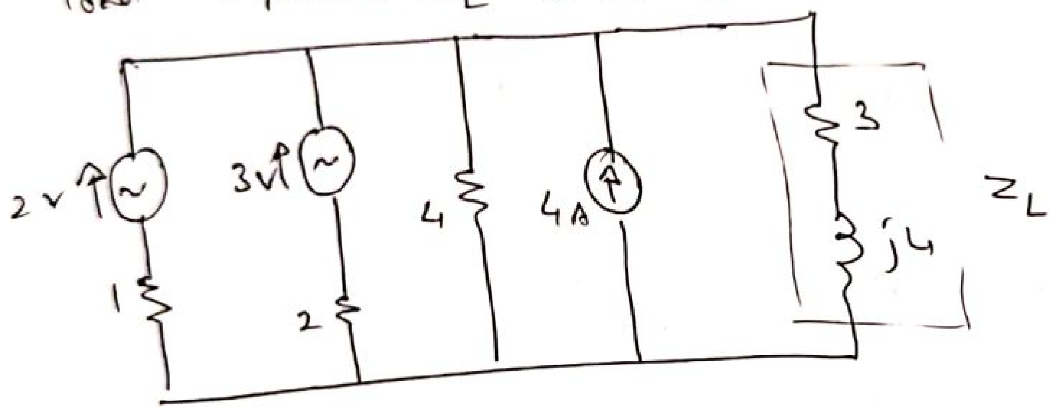
$$= \frac{22(1/5) + 48(1/12) + (-12)(1/4)}{1/5 + 1/12 + 1/4} = 10.13V$$

$$Z = \frac{1}{Y_1 + Y_2 + Y_3} = 1.88\,\Omega$$

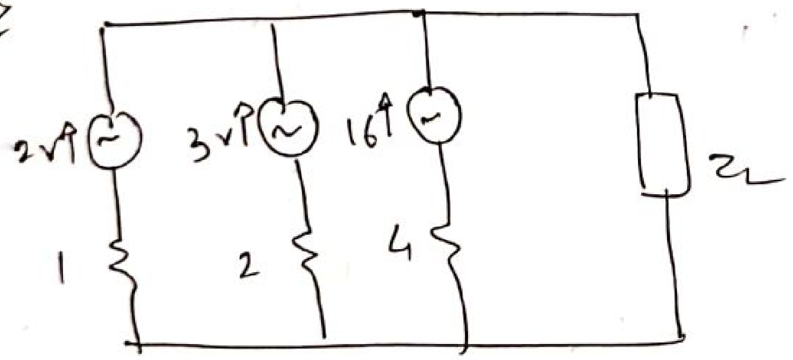



$$I = \frac{E}{Z+10} = 0.853\,A$$

3) Using Millman's theorem, find the current in the load impedance $Z_L$ in the ckt

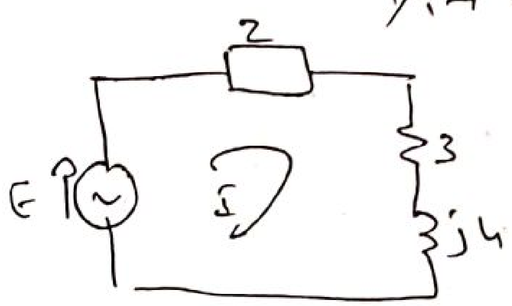



Sol/



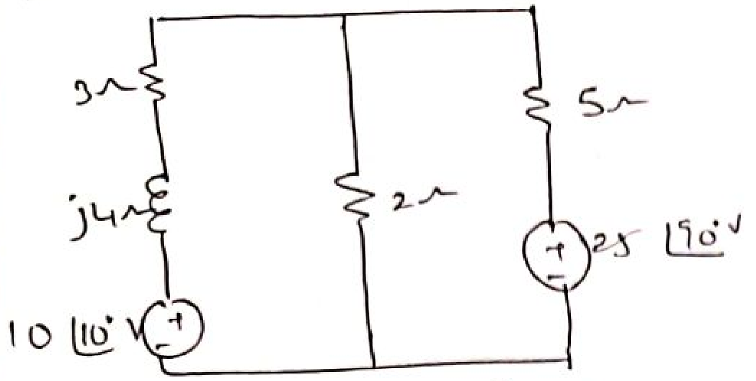

$$E = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3}{Y_1 + Y_2 + Y_3} = \frac{30}{7}\,V$$

$$Z = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{4}{7}\,\Omega$$




$$I = \frac{30/7}{4/7 + (3+j4)} = 0.8\angle{-48.25^\circ}\,A$$

4) Find $I_{2\Omega}$ using Millman's theorem (44)

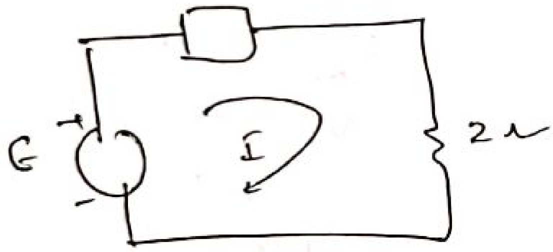



Sol

$$E = \frac{E_1 Y_1 + E_2 Y_2}{Y_1 + Y_2} = \frac{10\angle 10^\circ \left(\frac{1}{3+j4}\right) + 25\angle 90^\circ\left(\frac{1}{5}\right)}{\frac{1}{3+j4} + \frac{1}{5}}$$

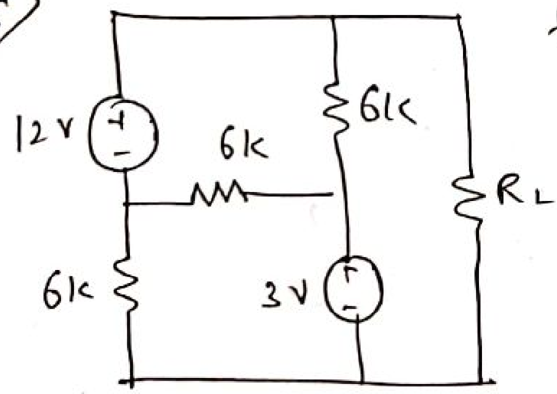
$$E = 10.06 \angle 97.12^\circ \text{ V}$$

$$Z = \frac{1}{Y_1 + Y_2} = \frac{1}{\frac{1}{3+j4} + \frac{1}{5}} = 2.8 \angle 26.56^\circ \ \Omega$$




$$I = 2.15 \angle 81.63^\circ \text{ A}$$

5)

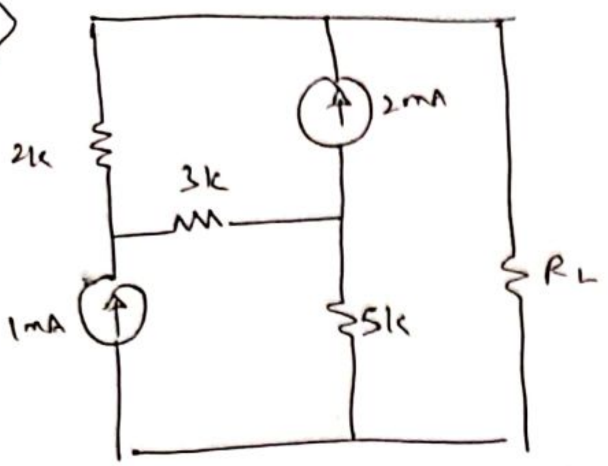



find $R_L$ for Max. power transfer & also find $P_{max}$.

Ans. $R_L = 2 k\Omega$

$P_m = 12.5 mW$

6)



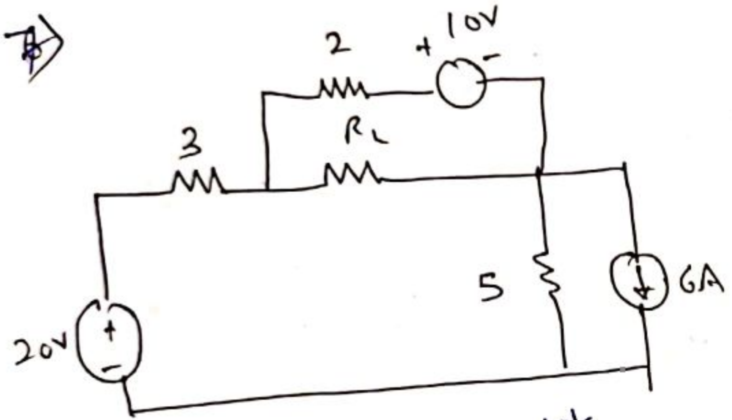

Max. Transfer Theorem

$R_L$? & $P_{max}$?

$R_L = 10k\Omega$

$P_m = 8.1mW$.

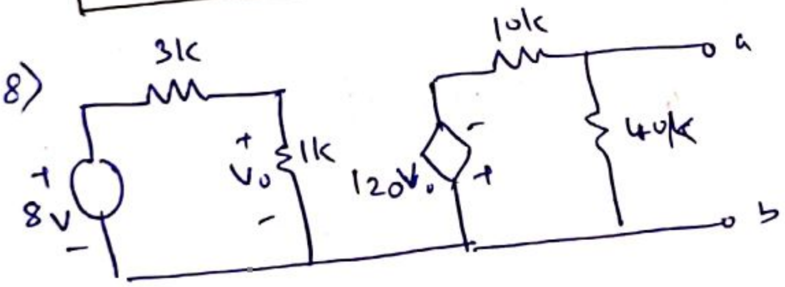
7)




$R_L = 1.6\Omega$

$P_{max} = 625$ mW.

8)



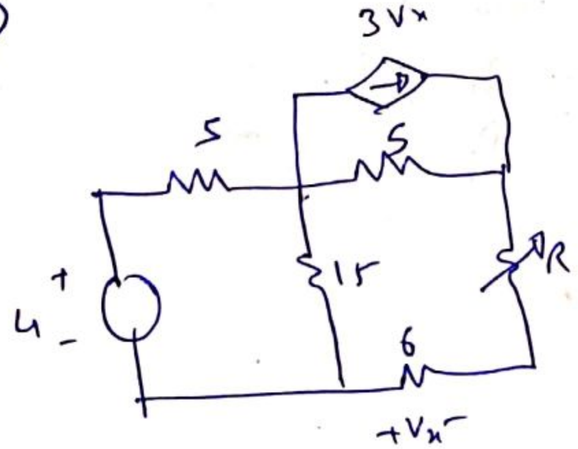

$P_{max} = 1.15W$

$V_{th} = -192V$

$I_{sc} = -24mA$

$R_{th} = 8k\Omega$

9)




R = ?

$P_{max}$ = ?

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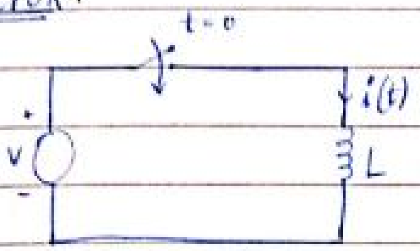
# MODULE - 3

## INITIAL CONDITIONS:

- **INDUCTOR:**




→ The switch is closed at $t=0$.

→ At $t=0^-$, corresponds to the instant when the switch is just open. [In the circuit shown above].

→ At $t=0^+$ is an instant when the switch is just closed.

The expression for current through inductor is:

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,dz$$

$$i = \frac{1}{L}\int_{-\infty}^{0^-} v\,dz + \frac{1}{L}\int_{0^-}^{t} v\,dz$$

$$i(t) = i(0^-) + \frac{1}{L}\int_{0^-}^{t} v\,dz$$

At $t=0^+$,

$$\Rightarrow i(0^+) = i(0^-) + \cancelto{0}{\frac{1}{L}\int_{0^-}^{0^+} v\,dz} \qquad [\text{Here } t=0^+]$$

$$\Rightarrow \boxed{i(0^+) = i(0^-)}$$

→ This equation means that the current in an inductor does not change instantaneously.

→ But Voltage may vary.



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- **INITIAL CONDITION EQUIVALENT OF AN INDUCTOR:**

| | $i(0^-)$ | NOTATION | EQUIVALENT @ $t = 0^+$ |
|---|---|---|---|
| 1) | $0$ <br> $[i(0^+) = 0]$ | | o.c. |
| 2) | $I_0$ <br> $i(0^+) = I_0$ | $I_0$ | $I_0$ |

- **FINAL OR STEADY STATE: [At $t = \infty$]**

The final condition equation of an inductor is derived from

$$\boxed{v = L\frac{di}{dt}}$$

* $v = 0$ ; $i =$ constant $\Rightarrow$ no resistance, thus, Short circuit.

- **FINAL CONDITION EQUIVALENT:**

| | NOTATION | EQUIVALENT @ $t = \infty$ |
|---|---|---|
| 1) | $i_L \neq 0$ | s.c. |
| 2) | $I_0$ | $\Rightarrow$ s.c. |



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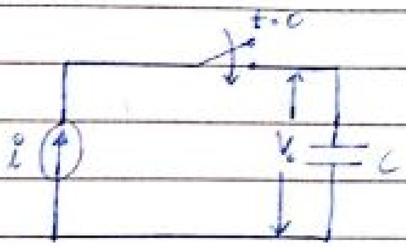
- ## CAPACITOR:

Expression for voltage across capacitor is:




$$V = \frac{1}{C}\int_{-\infty}^{t} i\,dz$$

$$V(t) = \frac{1}{C}\int_{-\infty}^{0^-} i\,dz + \frac{1}{C}\int_{0^-}^{t} i\,dz$$

$$V(0^+) = V(0^-) + \cancel{\frac{1}{C}\int_{0^-}^{t=0^+} i\,dz} \to 0 \qquad [\text{Here } t = 0^+]$$

$$\therefore \boxed{V(0^+) = V(0^-)}$$

→ Thus, the voltage across the capacitor does not change instantaneously.

→ But, current may vary.

- ## INITIAL CONDITION EQUIVALENT:

| | $V(0^-)$ | NOTATION | EQUIVALENT @ $t=0^+$ |
|---|---|---|---|
| i) | 0 $[V(0^+) = 0]$ |  |  S.C. |
| 2) | V $[V(0^+) = V]$ |  + − ←V→ |  + − V |

→ The final condition equivalent of the capacitor is derived from $\boxed{i = C\frac{dV}{dt}}$

→ At steady state $[t=\infty]$, $\frac{dV}{dt} = 0 \Rightarrow i = 0.$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## • FINAL CONDITION EQUIVALENT:

| | NOTATION | EQUIVALENT @ $t = \infty$ |
|---|---|---|
| 1) | | O.C |
| 2) | $+$ $-$ ; $V$ | $+$ $-$ $V$ O.C $\Rightarrow$ O.C. |

## • RESISTOR:

For a resistor, voltage is given by, $\boxed{V = iR}$. From this, we can observe that the current will change instantaneously if the voltage changes and vice versa.

## ** TABLE:

| Element | NOTATION | INITIAL CONDITION $t = 0^+$ | FINAL CONDITION (Steady State), $t = \infty$ |
|---|---|---|---|
| INDUCTOR | $L$ | O.C | S.C. |
| | $I_0$ ↳ already in steady state | $I_0$ | $\Rightarrow$ S.C. |
| CAPACITOR | $C$ | S.C. | O.C. |
| | $C$ $-$ $+$ $V_0$ ↳ Already in steady state | $-$ $+$ $V_0$ | $V_0$ $-$ $+$ O.C $\equiv$ O.C |



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- PROCEDURE FOR EVALUATING INITIAL CONDITIONS:

→ First, solve for initial values of currents and voltages, then, solve for derivatives.

→ For finding initial values of currents and voltages, an equivalent network of the original network at $\boxed{t = 0^+}$ is constructed according to the following rules:

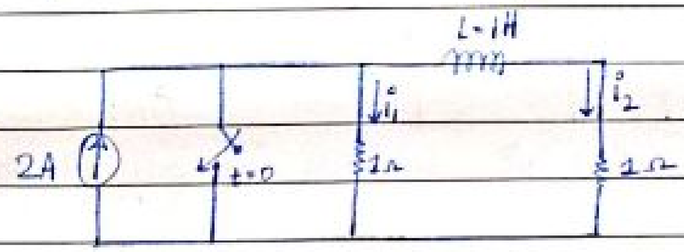
(1) Replace all inductors with O.C (or) current sources having value of current, flowing at $t = 0^+$.

(2) Replace all capacitors with S.C. (or) Voltage source of value $V_0 = \frac{Q_0}{C}$, if there is an initial charge.

(3) All resistors are left in the network without any changes.

PROBLEMS:

1) For the given circuit diagram, find $i_1(0^+)$ and $i_2(0^+)$.




S: The given circuit for $t<0$ is already in steady state. Therefore, the circuit is in steady state with the switch open at $t=0^-$. Thus, inductor is represented by Short Circuit.



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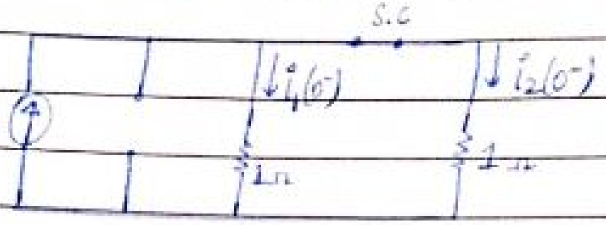
→ Equivalent circuit at $t = 0^-$ is:

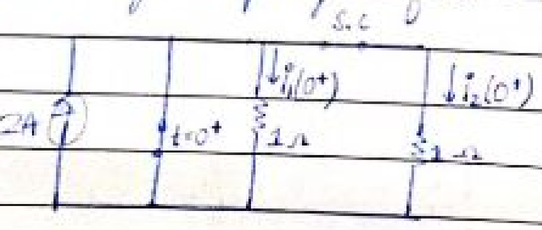



→ $i_2(0^-) = \frac{2 \times 1}{1+1} \Rightarrow \boxed{i_2(0^-) = 1A}$

→ $i_1(0^-) = \frac{2 \times 1}{1+1} \rightarrow \boxed{i_1(0^-) = 1A}$

→ At $t = 0^+$, the current through an inductor doesn't change instantaneously. Thus, $\boxed{i_2(0^+) = i_2(0^-) = 1A}$.

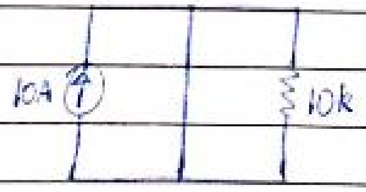
→ At $t = 0^+$, the switch is just closed, thus, there is no current flowing through the resistor [∵ 'I' chooses least resistant path → 'S.C']. Thus, $\boxed{i_1(0^+) = 0}$, i.e. ~~because~~ current in the resistor changes rapidly from 1A ⟶ 0A




Here, $i_1(t)$ changes as it is current through resistor only

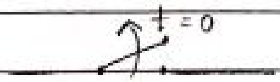
$i_2(t)$ doesn't change as it is current through inductor.

**NOTE:




→ Current through 10kΩ = 0

→ Current through S.C is maximum, as current chooses the least resistant path.

• 1)  → At $t = 0^-$, switch is closed



2)  → At $t = 0^-$, switch is open.



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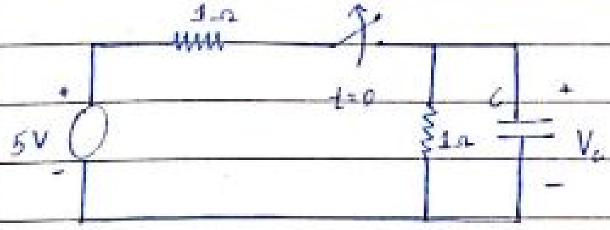
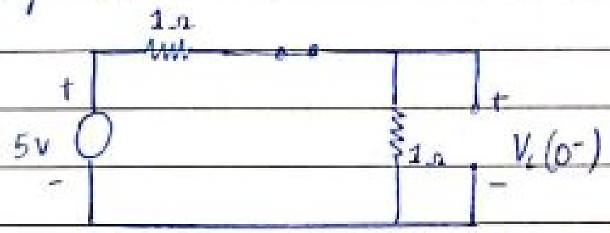
2) Find $V_c(0^+)$ such that the switch is closed for a long time.

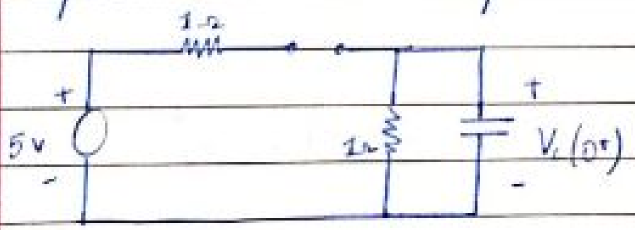



S: At $t=0^-$, switch is closed. Thus, the capacitor can be represented by O.C.




$$V_c(0^-) = \frac{5 \times 1}{1+1} \Rightarrow \boxed{V_c(0^-) = 2.5\,V}$$

→ At $t=0^+$, ~~but~~ the switch is open but voltage through the capacitor does not change instantaneously. $\Rightarrow V_c(0^+) = V_c(0^-) = 2.5\,V$




Here, the switch is just open. The voltage, $\boxed{V_c(0^+) = 2.5\,V}$

3) Find $i_L(0^+)$ and $V_c(0^+)$. Given, the circuit is in steady state with the switch in closed condition.

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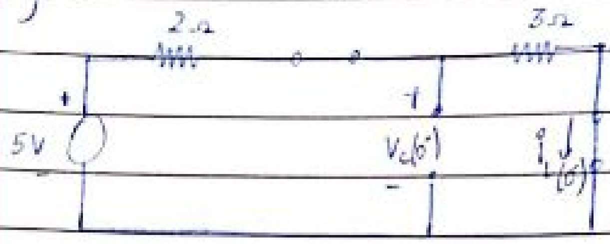
(8)

S: At $t=0^-$, the switch is closed [Steady State]. Thus, the capacitor is represented by O.C and Inductor is represented by S.C.




Applying KVL, $5 - 2i_1 - 3i_1 = 0$

$5 = 5i_1$

$i_L = \boxed{i_1 = 1A}$ ~~$i_{Total}$~~

The current does not change instantaneously, i.e, $i_L(0^-) = i_L(0^+)$

~~$\therefore \boxed{i_L(0^+) = 1A}$~~

→ ~~At $t=0^+$, the switch is open.~~

~~$i_L(3\Omega) = \frac{1 \times 2}{2+3} = \frac{2}{5}$~~

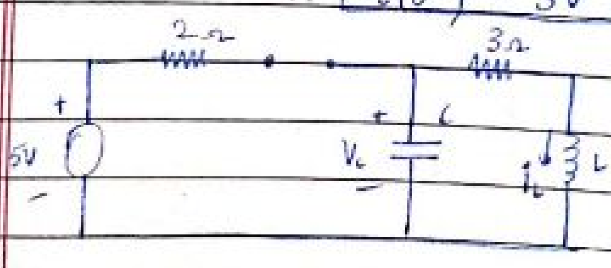
~~$\boxed{i_L(0^+) = \frac{2}{5}A}$~~

$\boxed{i_L(0^+) = 1A}$

$V_c(0^-) = \frac{5 \times 3}{2+3} = \frac{5 \times 3}{5} \rightarrow \boxed{V_c(0^-) = 3V}$

→ At $t=0^+$, the switch is open; the voltage does not change instantaneously. Thus, $V_c(0^+) = V_c(0^-)$.

$\therefore \boxed{V_c(0^+) = 3V}$



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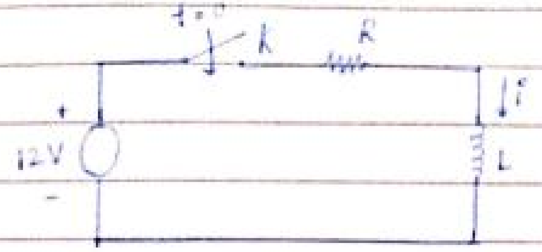
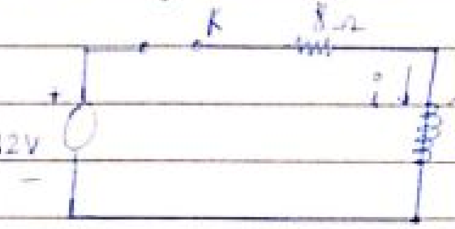
(9)

4) Switch, K is closed at $t=0$, with zero current in the inductor. Find $i$, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$. Given, $R = 8\,\Omega$ & $L = 0.2H$.

S:



At $t=0^-$, the switch is open. & ~~inductor is represented by s.c.~~



Remember, don't consider switch was open before closing!

~~$i = \frac{V}{R} = \frac{12}{8}$~~ $\boxed{i = 0\,A}$

~~$i = 1.5$~~

The current does not change instantaneously. Thus, $i(0^+) = i(0^-)$

$\therefore \boxed{i(0^+) = 0}$.

→ At $t = 0^+$, switch is closed. The current through inductance will not change instantaneously. Thus, $i(0^+) = i(0^-) = 0$.

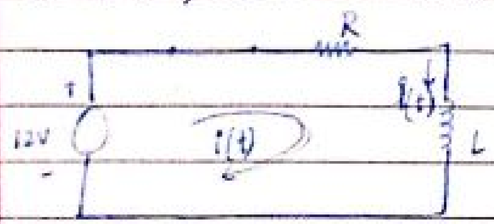
NOTE: i) $V_c = \frac{1}{C}\int_0^t i(z)\,dz$

ii) $V_L = L\frac{di}{dt}$

* We can't take $-L\,i(t)$ coz it is pure inductance.

$t > 0$

→ At $t = 0^+$, switch is closed.



Applying KVL,

Voltage across inductor

$12 - 8i(t) - L\frac{di(t)}{dt} = 0$

$\boxed{8i(t) + 0.2\frac{di(t)}{dt} = 12} \longrightarrow (1)$



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(10)

At $t=0^+$, $\cancel{8i(0^+)} + 0.2\frac{di(0^+)}{dt} = 12$

$$\frac{di(0^+)}{dt} = \frac{12}{0.2}$$

$$\boxed{\frac{di(0^+)}{dt} = 60\ A/s}$$ Ampere/second.

Differentiating eq ① w.r.t t.

$$+8\,i(t) + 0.2\frac{di(t)}{dt} = 12$$

$$\boxed{8\frac{di(t)}{dt} + 0.2\frac{d^2i(t)}{dt^2} = 0}$$

At $t=0^+$, $8\frac{di(0^+)}{dt} + 0.2\frac{d^2i(0^+)}{dt^2} = 0$

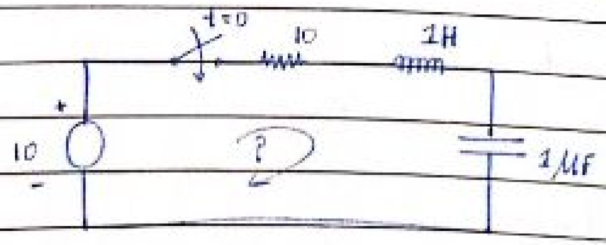
$$8(60) + 0.2\frac{d^2i(0^+)}{dt^2} = 0$$

$$\therefore \frac{d^2i(0^+)}{dt^2} = \frac{-480}{0.2}$$

$$\boxed{\frac{d^2i(0^+)}{dt^2} = -2400\ A/s^2}$$

5) Switch is closed at $t=0$. Find $i$, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$.

S:



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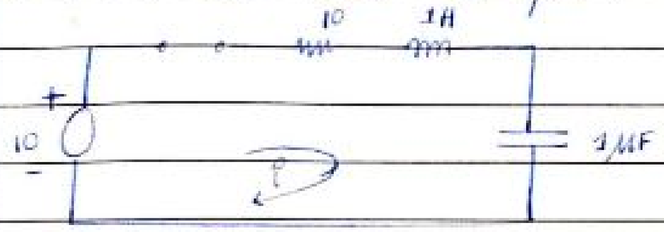
(11)



→ At $t=0^-$, the switch is open.

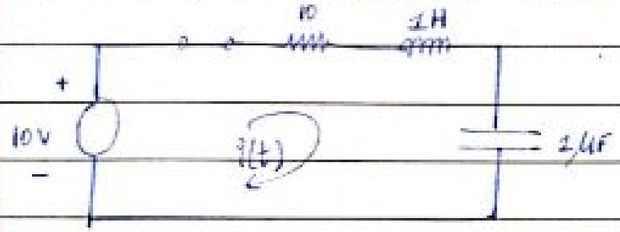



$\boxed{V_c(0^-)=0} = V_c(0^+)$

$i(0^-) = 0.$

At $t=0^+$, The current doesn't change instantaneously. Thus, $\boxed{i(0^-) = i(0^+) = 0}$.

→ At $t=0^+$, the switch is closed.




Applying KVL, $\boxed{10 - 10\,i(t) - 1\frac{di(t)}{dt} - \frac{1}{1\mu F}\int i(t)\,dt = 0} \longrightarrow (1)$

$\Rightarrow \boxed{10\,i(t) + \frac{di(t)}{dt} + V_c(t) = 10} \longrightarrow (2)$

→ At $t=0^+$, $\cancel{10\,i(0^+)}^{0} + \frac{di(0^+)}{dt} + \cancel{V_c(0^+)}^{0} = 10$

$\therefore \boxed{\frac{di(0^+)}{dt} = 10}\ A/s$

Differentiate eq. (1) w.r.t 't'.

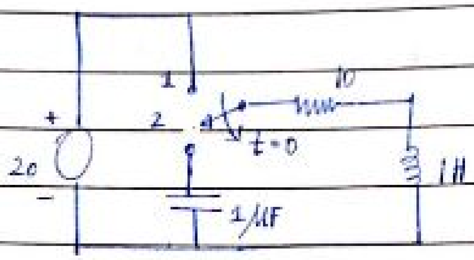
$10\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + \cancel{\frac{1}{1\mu F}\,i(t)}^{0} = 0$

$\cancel{10(10)} \cancel{+ di}$

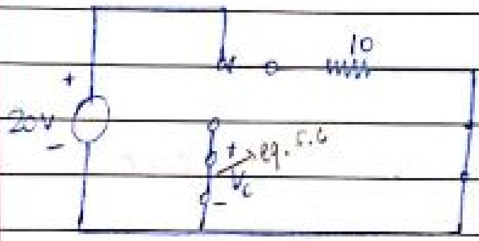
→ At $t=0^+$, $10(10) + \frac{d^2i(0^+)}{dt} + \cancel{\frac{1}{10^{-6}}} = 0$

$\boxed{\frac{d^2i(0^+)}{dt} = -100\ A/s^2}$

6) 'K' is changed from position 1 to position 2 at $t=0$, steady state condition has been reached at position 1. Calculate $i$, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$.




S: At $t=0^-$, the switch is at position 1. Capacitor is O.C. Inductor is S.C.



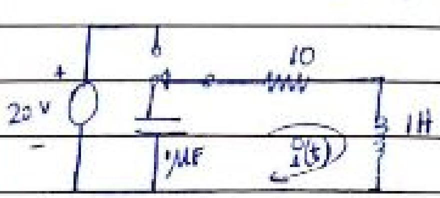

$$\boxed{V_c(0^+) = V_c(0^-) = 0}$$

$$i(0^-) = \frac{V}{R} = \frac{20}{10} = 2$$

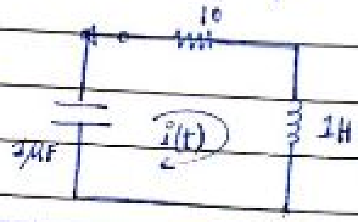
$$\boxed{i(0^-) = 2A}$$

At $t=0^+$, the voltage and current do not change instantaneously.
Thus, $\boxed{i(0^+) = i(0^-) = 2A}$ ; $\boxed{V_c(0^+) = V_c(0^-) = 0}$.

→ When switch changes from position 1 to 2; $[At\ t=0]$




⟹




Apply KVL, $\boxed{\frac{-1}{1\mu F}\int_0^t i(t)\,dt - 10i(t) - 1\frac{di(t)}{dt} = 0} \rightarrow (1)$

$$\boxed{\frac{1}{\mu F}\int_0 i(t)\,dt + 10i(t) + \frac{di(t)}{dt} = 0}$$

At $t = 0^+$,

$$\cancel{V_c(0^+)}^{\,0} + 10\, i(0^+) + \frac{di(0^+)}{dt} = 0$$

$$-10(2) = \frac{di(0^+)}{dt}$$

$$\therefore \boxed{\frac{di(0^+)}{dt} = -20\ A/s}$$

Differentiate eq. (1) w.r.t 't'.

$$\frac{1}{\mu F}\, \partial i(t) + 10\frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} = 0$$

$$\frac{1}{\mu F}\, i(t)\, \cancel{V_c(t)} + 10\frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} = 0$$

At $t = 0^+$, $\cancel{V_c(0^+)} + 10\frac{di(0^+)}{dt} + \frac{d^2 i(0^+)}{dt^2} = \cancel{0} + \frac{1}{\mu F}\, i(0^+) = 0$

$$10(-20) + \frac{d^2 i(0^+)}{dt} + 2\times10^6 = 0$$

$$\frac{d^2 i(0^+)}{dt} = 200 - 2\times10^6$$

$$\frac{d^2 i(0^+)}{dt} = -1.9998\times10^6 \simeq -2\times10^6$$

$$\boxed{\frac{d^2 i(0^+)}{dt} \simeq -2 MA/s^2}$$



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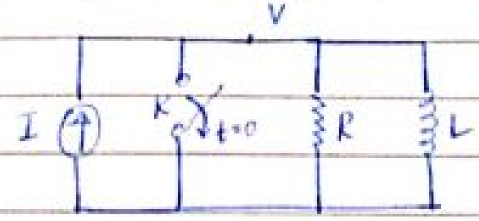
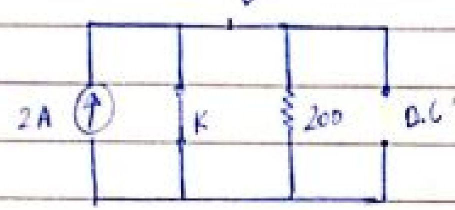
(14)

10) 'K' is opened at $t=0$. At $t=0^+$, solve for $V$, $\frac{dV}{dt}$, $\frac{d^2V}{dt^2}$ if $I=2A$, $R=200\Omega$, $L=1H$. Further, switch is open at $t=0^+$ & closed at $t=0^-$.




S: At $t=0^-$,




(∵) the current flows through S.C. Hence 'I' through 'L' is 0 ⇒ O.C.

→ Voltage through 'K' and current through 'L' do not change instantaneously.

Thus $V(0^-) = 0 \neq V(0^+)$

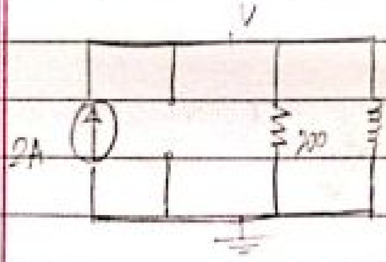
$\boxed{i_L(0^-) = i_L(0^+) = 0}$ [∵ O.C]

At $t=0^+$ switch is open but 'L' is O.C only.

$\Rightarrow V = I \times R = 2 \times 200$

$\boxed{V(0^+) = 400V}$ ← (2)

At $t=0^+$




KCL at (V):

$$\boxed{0 = -2 + \frac{V}{200} + \frac{1}{L}\int_0^t V_L(t)\,dt} \rightarrow (1)$$

$$\frac{V(t)}{200} - 2 + i_L(t) = 0$$

At $t=0^+$ $\quad \frac{V(0^+)}{200} - 2 + \cancel{i_L(0^+)}^{\,0} = 0$

$$V(0^+) = 2 \times 200$$

$$\boxed{V(0^+) = 400V}$$



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15

Diff. ① w.r.t

$$\cancel{-2}^{0} + \frac{1}{200}\frac{dV(t)}{dt} + \frac{1}{1}V(t) = 0 \longrightarrow ②$$

At $t = 0^+$,

$$\frac{1}{200}\frac{dV(0^+)}{dt} + V(0^+) = 0$$

$$\frac{dV(0^+)}{dt} = -400 \times 200$$

$$\boxed{\frac{dV(0^+)}{dt} = -80000 \text{ V/s}}$$

Diff eq. ② w.r.t t

$$\frac{1}{200}\frac{d^2V(t)}{dt^2} + \frac{dV(t)}{dt} = 0$$

At $t = 0^+$ $$\frac{1}{200}\frac{d^2V(0^+)}{dt^2} + \frac{dV(0^+)}{dt} = 0$$

$$\frac{d^2V(0^+)}{dt^2} = -(-80000) \times 200$$

$$\boxed{\frac{d^2V(0^+)}{dt^2} = 16 \text{ MV/s}^2}$$



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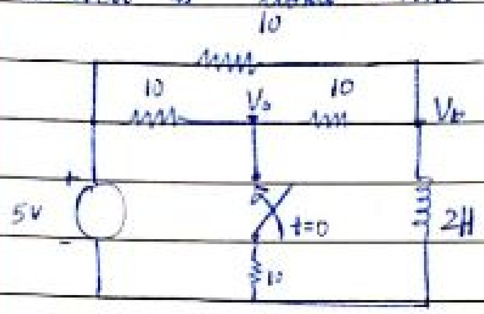
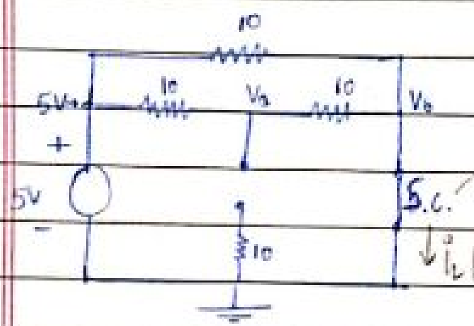
12) A steady state is reached with switch 'K' open. At t=0, switch is closed. Find $V_a(0^+)$ & $V_b(0^+)$.




S: At $t=0^-$

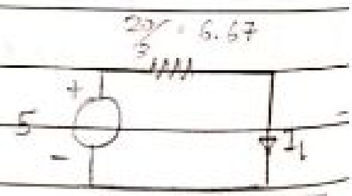



The other loop current can flow through the inductor. So, it's been in a steady state. ⇒ S.C

Apply KCL at $V_a$:

$$\frac{V_a-5}{10}+\frac{V_a-V_b}{10}=0$$

$$\boxed{V_a\left[\frac{1}{10}+\frac{1}{10}\right]-V_b\left[\frac{1}{10}\right]=\frac{5}{10}} \rightarrow ①$$




$\boxed{I_L = 0.75A}$

KCL @ $V_b$:

$$\frac{V_b-V_a}{10}+\frac{V_b-5}{10}+i_L(t)=\cancel{0}+0.75=0$$

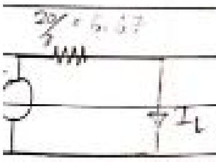
$$\boxed{-V_a\left(\frac{1}{10}\right)+V_b\left[\frac{1}{10}+\frac{1}{10}\right]=\cancel{2}\ 1.25} \rightarrow ②$$

$\boxed{V_a(0^-)=7.5\ V}$

$\boxed{V_b(0^-)=10\ V}$

At $t = 0^-$,

(low through the
a steady state.

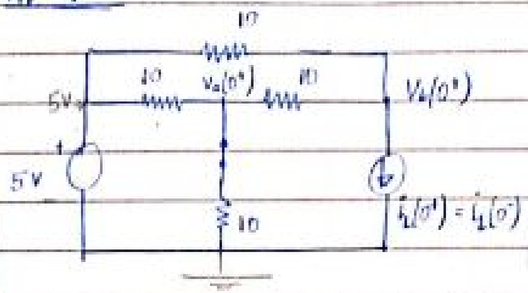


$\boxed{I_L = 0.75A}$

(13)

At $t = 0^+$

Replace inductor



KCL @ $V_a$

$$\frac{V_a - 5}{10} + \frac{V_a - V_b}{10} + \frac{V_a}{10} = 0$$

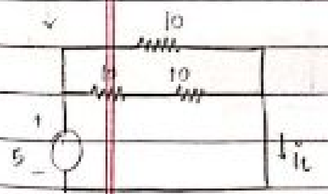
$$\boxed{V_a\left[\frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right] - V_b\left(\frac{1}{10}\right) = \frac{1}{2}} \rightarrow (1)$$

KCL @ $V_b$.

$$\frac{V_b - 5}{10} + \frac{V_b - V_a}{10} + i_L(0^-) = 0$$

$$\boxed{-V_a\left(\frac{1}{10}\right) + V_b\left(\frac{1}{10} + \frac{1}{10}\right) = -0.25} \rightarrow (2)$$

$\boxed{V_a(0^+) = 1.5V}$ $\boxed{V_b(0^+) = -0.5V}$



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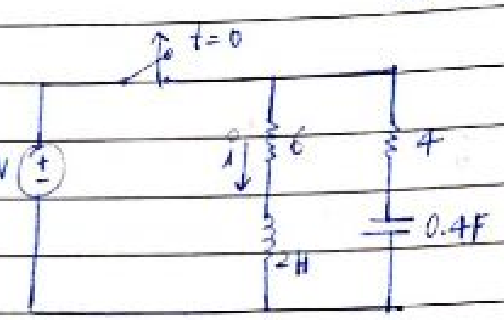
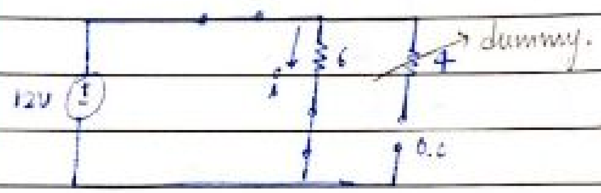
(19)

9-10-19 1) Find $i(0^+)$ and $V(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dV(0^+)}{dt}$, $i(\infty)$ & $V(\infty)$, $\frac{d^2 i(0^+)}{dt^2}$




S: At $t=0^-$, switch is closed.




$\rightarrow$ $i(0^-) = \frac{12}{6}$

$\boxed{i(0^-) = 2A}$

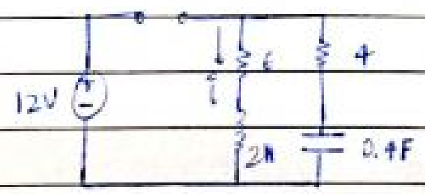
$\rightarrow$ $\boxed{V(0^-) = 12V}$

$\rightarrow$ Since current through inductor and voltage across capacitor do not change instantaneously.

$i(0^-) = i(0^+) = 2A$

$V(0^-) = V(0^+) = 12V$

At $t > 0$,

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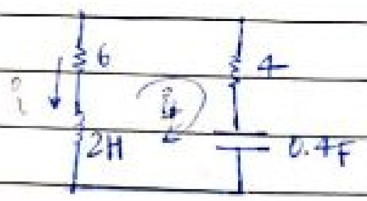


Applying KVL,

$$+\cancel{6}6i(t) - 4\,i(t) - 2\frac{di(t)}{dt} - \underbrace{\frac{1}{0.4}\int_0^t i(t)\,dt}_{V_c} = 0 \quad \text{(1)}$$

-ve current direction is opp.

~~At $t=0^+$~~ $\cancel{-10\,i(0^+)}$ $\cancel{-2\frac{di(0^+)}{dt}}$ $\cancel{-\frac{1}{0.4}}$



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classmate Date Page

$$\boxed{\frac{di(0^+)}{dt} = -4\ A/s}$$ Check

$\Rightarrow \boxed{+2i(t) - V_c(t) = 2\frac{di(t)}{dt}} \longrightarrow (2)$

$\Rightarrow +1i(t) - \frac{V_c(t)}{2} = \frac{di(t)}{dt}$

At $t=0^+$ $\frac{di(0^+)}{dt} = +(2) - \frac{12}{2} = 2-6$

$\therefore \boxed{\frac{di(0^+)}{dt} = -4\ A/s}$

$\rightarrow$ Diff eq (1) w.r.t 't'

$$2\frac{di(t)}{dt} - 2\frac{d^2i(t)}{dt^2} - \frac{1}{0.4}i(t) = 0$$

At $t=0^+$, $2\frac{di(0^+)}{dt} - 2\frac{d^2i(0^+)}{dt^2} - \frac{1}{0.4}i(0^+) = 0$

$$2(-4) - \frac{1}{0.4}(2) = 2\frac{d^2i(0^+)}{dt^2} \Rightarrow \frac{-8-5}{2} = -6.5$$

$\Rightarrow \boxed{\frac{d^2i(0^+)}{dt^2} = -6.5\ A/s^2}$

$\rightarrow$ Differentiate eq (2) w.r.t 't'

$$+2\frac{di(t)}{dt} - \frac{dV_c(t)}{dt} = 2\frac{d^2i(t)}{dt^2}$$

At $t=0^+$,

$$\frac{dV_c(0^+)}{dt} = \cancel{2(22.5)} + \cancel{10(-4)} \cancel{= 45-40}$$

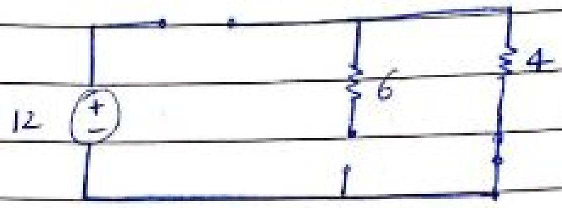
$$= 2\frac{di(0^+)}{dt} - 2\frac{d^2i(0^+)}{dt^2} = -8+13 = \underline{5}$$

$\therefore \boxed{\frac{dV(0^+)}{dt} = 5\ V/s}$

(*) $i_c = C\frac{dV}{dt}$

$\Rightarrow \frac{dV(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{-2}{0.4} = -5$ current direction

For $t = \infty$, switch is open.

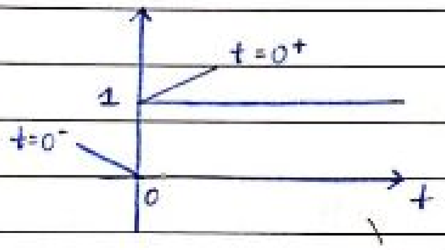


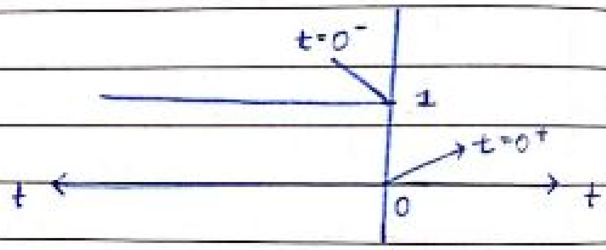

$\boxed{i(\infty) = 0}$ ; $\boxed{V(\infty) = 0}$

10-10-19

- 3 TYPES OF FUNCTION

1) UNIT STEP INPUT:

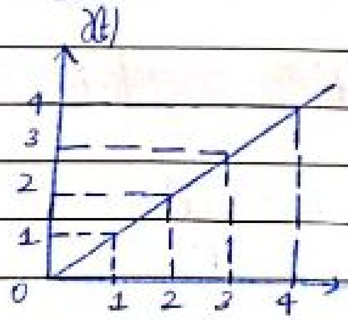



$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t \leq 0 \end{cases}$$

$$u(-t) = \begin{cases} 0 & t \geq 0 \\ 1 & t \leq 0 \end{cases}$$

2) RAMP INPUT:




$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t \leq 0 \end{cases}$$



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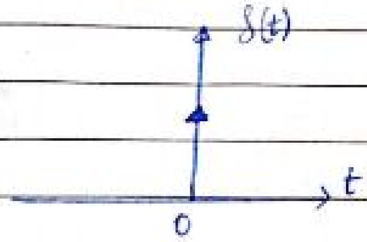
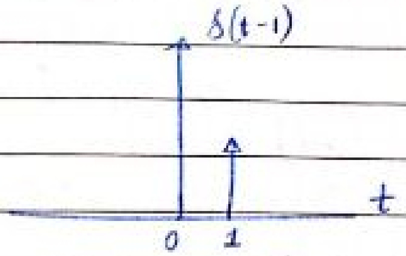
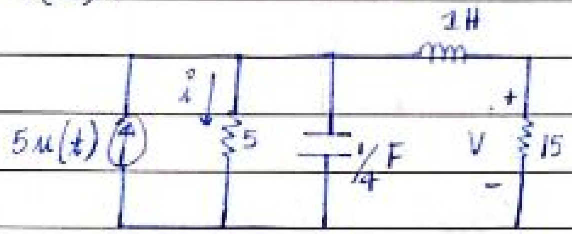
## 3) Impulse Input :



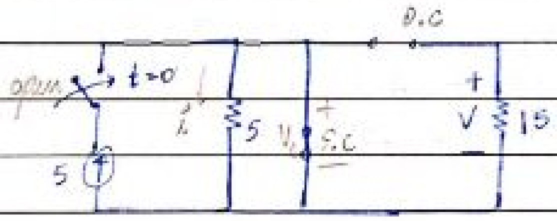

1) For the given circuit, find $i(0^+)$, $v(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$, $i(\infty)$, $v(\infty)$.

☆ Check



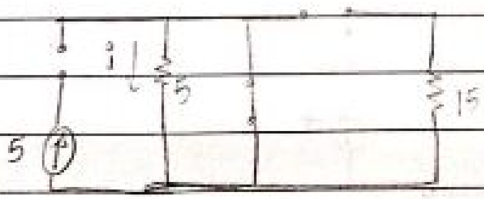

S: At $t = 0^-$,

Switch is open:




$\boxed{i(0^-) = 0}$

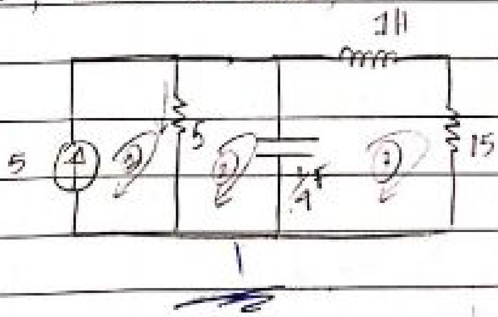
$V_c(0^-) = 0$




$\boxed{i(0^-) = i(0^+) = 0}$

$\boxed{V_c(0^-) = V_c(0^+) = 0}$

At $t = 0$, switch is closed




$\frac{25 \times 15}{20}$ $\frac{3}{4}$

$i = \frac{V_c}{5}$

For $u(t)$, $t = 0^-$, switch open, $t = 0$, s - closed

For $u(-t)$, $t = 0^-$, SW - closed, $t = 0$ S - open



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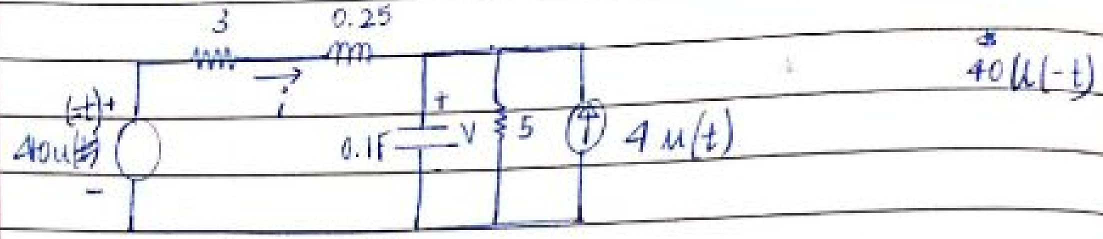
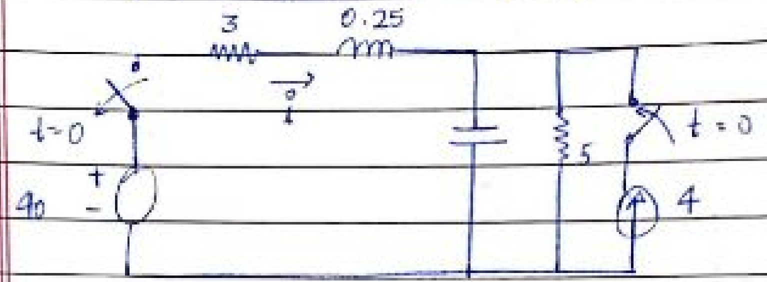
(23)

2) For the given circuit $V(0^+)$, $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$, $i(\infty)$, $v(\infty)$




S:

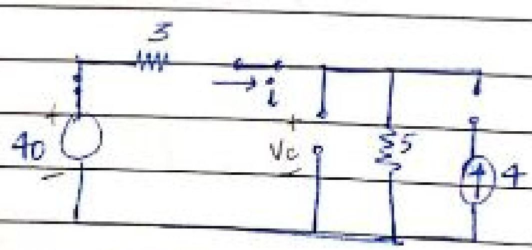



* → 'i' is the current through inductor and 'v' is the voltage across the capacitor. These values do not change instantaneously

→ $i(0^-) = i(0^+) = 5A$ } calculations shown below.

→ $v(0^+) = v(0^-) = 25V$

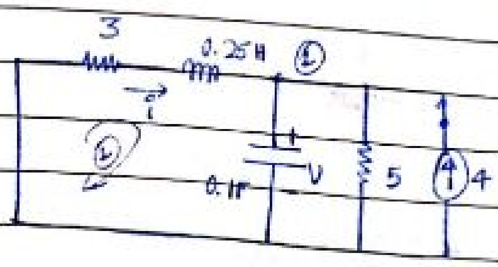
- At $t = 0^-$,




→ $i(0^-) = \frac{40}{5+3}$ ⇒ $\boxed{i(0^-) = \cancel{8}\ 5A}$

→ $V(0^-) = \frac{40 \times 5}{8}$ ⇒ $\boxed{V(0^-) = 25\,V}$

- At $t > 0$,




$40u(-t) = 0$
↳ S.C.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Applying KVL, loop ①

$$-3i(t) - L\frac{di(t)}{dt} - V_c = 0$$

$$-3i(0^+) - L\frac{di(0^+)}{dt} - V_c(0^+) = 0 \qquad [\text{At } t = 0^+]$$

$$-3(5) - 0.25\frac{di(0^+)}{dt} - 25 = 0$$

$$\therefore \boxed{\frac{di(0^+)}{dt} = -160\ A/s}$$

→ $\left\{ i_c = C\frac{dV}{dt} \right\}$ → This cannot be used coz same current does not flow.

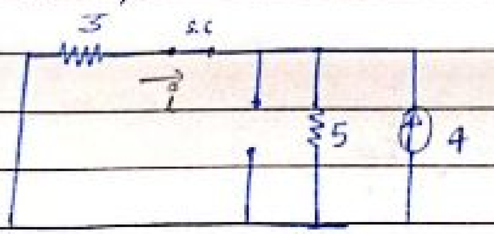
Applying KCL at node ①:

$$-i - 4 + C\frac{dV}{dt} + \frac{V}{5} = 0$$

At $t = 0^+$; $$-5 - 4 + 0.1\frac{dV(0^+)}{dt} + \frac{25}{5} = 0$$

$$\therefore \boxed{\frac{dV(0^+)}{dt} = 40\ V/s}$$

→ for $t = \infty$,




→ $i(\infty) = -\frac{4 \times 5}{8}$ — -ve bcoz of current direction

$$\therefore \boxed{i(\infty) = -2.5\ A}$$

$$V(\infty) = 4 \times \frac{3 \times 5}{8} = iR$$

$$V(\infty) = \frac{15}{2}$$

$$\therefore \boxed{V(\infty) = 7.5\ V}$$



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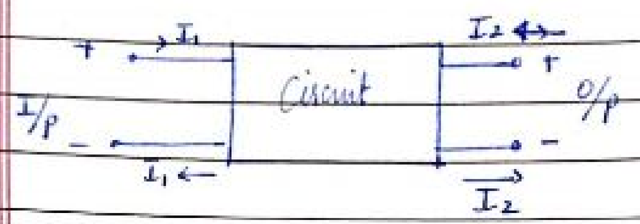
① 

10-10-19

# MODULE - 5

## Two Port Parameters




→ A pair of terminals through which current may enter or leave a network is known as a port.

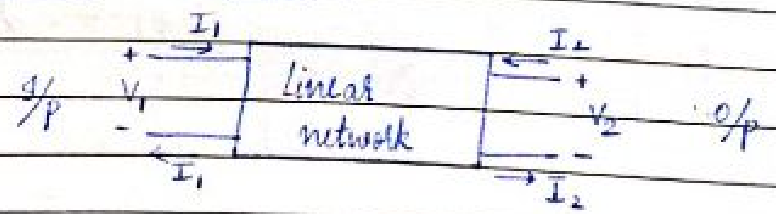
→ A port is an access to the network and consists of a pair of terminals.

→ A two port network has two ports, one of which is the input and the other is the output.

• Types of 2 Parameters [2- Port]

① Admittence (or) Y/S.C. parameter
② Impedence / Z / O.C. parameter
③ Hybrid / H parameter
④ Transmission / ABCD parameter.

• ADMITTENCE PARAMETER:




→ Consider the network shown. It is assumed that the network is linear and contains no dependent sources.

→ Considering the principle of superposition, current $I_1$ can be written as sum of 2 components, one due to $V_1$ & the other due to $V_2$

* $Y_{12} = Y_{21}$ [When no dependent sources are there]





We have, $\boxed{I_1 = Y_{11}V_1 + Y_{12}V_2}$ and $\boxed{I_2 = Y_{21}V_1 + Y_{22}V_2}$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where, $Y_{11} = \frac{I_1}{V_1}\Big|_{V_2 = 0\,[S.C.]}$ → i/p admittance / driving point admittance at the i/p port.

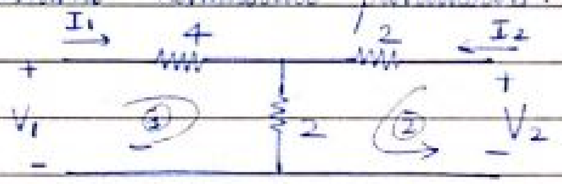
$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1 = 0\,[S.C.]}$ → Reverse transfer admittance.

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2 = 0}$ → Forward transfer admittance.

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1 = 0}$ → o/p admittance / driving point admittance at the o/p port.

PROBLEMS:

1) Determine admittance parameters.




* $I_1$ → Clockwise
$I_2$ → Anti-clockwise } Fixed.
→ If there is a loop in between, any direction can be considered.

Standard admittance equations are:

$$\boxed{I_1 = Y_{11}V_1 + Y_{12}V_2} \longrightarrow (1)$$
$$\boxed{I_2 = Y_{21}V_1 + Y_{22}V_2} \longrightarrow (2)$$

Applying KVL to loop (1):

$$V_1 - 4I_1 - 2(I_1 + I_2) = 0$$
$$\boxed{V_1 = 6I_1 + 2I_2} \longrightarrow (3)$$

KVL to loop (2):

$$-2I_2 - 2(I_1 + I_2) + V_2 = 0$$
$$\boxed{V_2 = 2I_1 + 4I_2} \longrightarrow (4).$$



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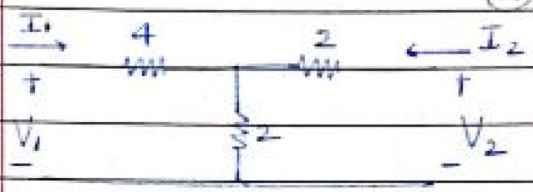
Thus, $\begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

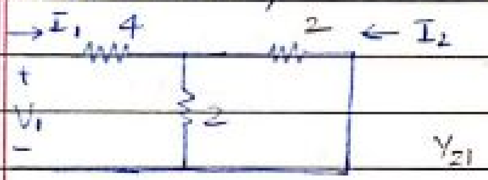
Thus; $\boxed{Y_{11} = 0.2\,\mho}$ ; $\boxed{Y_{12} = -0.1\,\mho}$

$\boxed{Y_{21} = -0.1\,\mho}$ ; $\boxed{Y_{22} = 0.3\,\mho}$

(d)




→ When $V_2 = 0$, $Y_{11} = \dfrac{I_1}{V_1}$ ; $R = \left[4 + \dfrac{2\times 2}{2+2}\right] = 4+1 = 5$




$\therefore Y_{11} = 1/5 \Rightarrow \boxed{Y_{11} = 0.2\,\mho}$

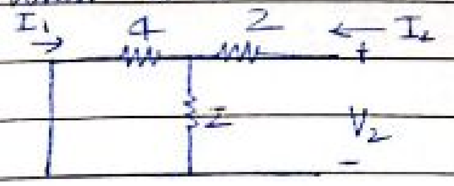
$Y_{21} = \dfrac{I_2}{V_1}$ ; $I_2 = -\dfrac{I_1 \times 2}{2+2} \Rightarrow \boxed{I_2 = -\dfrac{I_1}{2}}$

$$\Rightarrow Y_{21} = \frac{-I_1}{2V_1} = \frac{-\cancel{V_1}}{2(5\cancel{V_1})} = \frac{-1}{10}$$

$\boxed{Y_{21} = -0.1\,\mho}$

$\boxed{Y_{21} = Y_{12} = -0.1\,\mho}$ --> When no dependent source are there.

→ When $V_1 = 0$




$Y_{22} = \dfrac{I_2}{V_2}$ ; $R = \left[2 + \dfrac{4\times 2}{4+2}\right] = 3.33$

$\Rightarrow Y_{22} = \dfrac{1}{R} \Rightarrow \boxed{Y_{22} = 0.3\,\mho}$

$Y_{12} = \dfrac{I_1}{V_2}$ ; $I_1 = \dfrac{-I_2 \times 2}{4+2} \Rightarrow \boxed{I_1 = -\dfrac{I_2}{3}}$ | $V_2 = 3.3\,I_2$

$$Y_{12} = \frac{-I_2}{3V_2} = \frac{-\cancel{I_2}}{3\times(3.3)\cancel{I_2}} \simeq -0.1\,\mho$$



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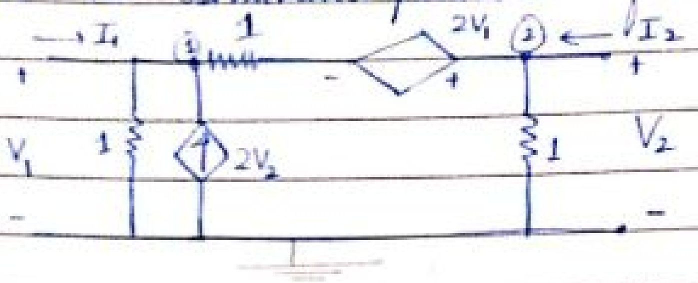
2) Find the admittance parameters for the network shown.




S: Standard equations are:

$$Y_{11}V_1 + Y_{12}V_2 = I_1 \longrightarrow (1)$$
$$Y_{21}V_1 + Y_{22}V_2 = I_2 \longrightarrow (2)$$

KCL at (1):

$$-I_1 + \frac{V_1}{1} - 2V_2 + \frac{V_1 - (-2V_1) + -V_2}{1} = 0$$

$$\boxed{V_1[1+1+2] - V_2[2+1] = I_1} \longrightarrow (3)$$

KCL at (2):

$$-I_2 + \frac{V_2}{1} + \frac{V_2 - V_1 - 2V_1}{1} = 0$$

$$\boxed{-V_1[1+2] + V_2[1+1] = I_2} \longrightarrow (4)$$

Comparing (1) & (3):

$$\boxed{Y_{11} = 4\,\Omega^{-1}} \; ; \; \boxed{Y_{12} = -3\,\Omega^{-1}}$$

Comparing (2) & (4):

$$\boxed{Y_{21} = -3\,\Omega^{-1}} \; ; \; \boxed{Y_{22} = 2\,\Omega^{-1}}$$



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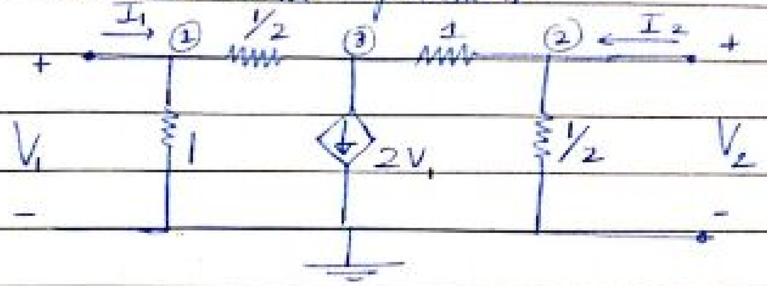
(6)

3) Find the admittence parameters.

① & ② are fixed.




S: Standard equations are:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \longrightarrow ①$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \longrightarrow ②$$

KCL @ node ①:

$$-I_1 + \frac{V_1}{1} + 2(V_1 - V_3) = 0$$

$$\boxed{V_1[1+2] - 2V_3 = I_1} \longrightarrow ③$$

Subs. ④

$$\boxed{3V_1 - \frac{2V_2}{3} = I_1} \to ⑥$$

KCL @ ③:

$$2(V_3 - V_1) + 2V_1 + \frac{V_3 - V_2}{1} = 0$$

$$\cancel{-2V_1} \cancel{+2V_1} - V_2 + 3V_3 = 0$$

$$\boxed{\cancel{-4V_1} - V_2 + 3V_3 = 0} \longrightarrow ④$$

$$\hookrightarrow \boxed{V_3 = \frac{V_2}{3}}$$

KCL @ ~~④~~ ②:

$$-I_2 + 2V_2 + \frac{V_2 - V_3}{1} = 0$$

$$\boxed{V_2(2+1) - V_3 = I_2} \longrightarrow ⑤$$

$$3V_2 - \frac{V_2}{3} = I_2$$

$$\Rightarrow V_2\left[3 - \frac{1}{3}\right] = I_2$$

$$\boxed{\frac{8}{3}V_2 = I_2} \longrightarrow ⑦$$



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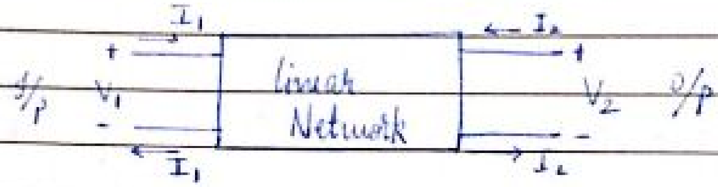
Comparing ③ & ⑥ :

$\boxed{Y_{11} = 3\ \Omega^{-1}}$ $\boxed{Y_{12} = -2/3\ \Omega^{-1}}$

Comparing ② & ⑦ :

$\boxed{Y_{21} = 0}$ $\boxed{Y_{22} = 8/3\ \Omega^{-1}}$

- IMPEDENCE PARAMETERS:




$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

where, $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ ⟶ Input impedence when the output is open. circuit ($\because I_2 = 0$)

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ ⟶ Reverse transfer impedence when input is O.C [$\because I_1 = 0$]

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ ⟶ Forward transfer impedence when output is O.C [$\because I_1 = 0$]

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$ ⟶ Output impedence when input is O.C [$\because I_1 = 0$]

NOTE: $\boxed{Z_{12} = Z_{21}}$ if there are no dependent sources



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(8)

$V_1 = Z_{11} I_1 + Z_{12} I_2$
$V_2 = Z_{21} I_1 + Z_{22} I_2$

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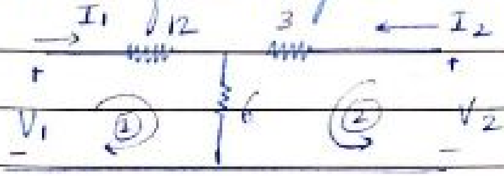
1) Find Z-parameters for the given circuit.




S: A Standard equations [Impedance parameter eqs] are:

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$$

Applying KVL to loop (1):

$$V_1 - 12 I_1 - 6(I_1 + I_2) = 0$$
$$\boxed{V_1 = 18 I_1 + 6 I_2} \longrightarrow (3)$$

Applying KVL to loop (2):

$$-3 I_2 - 6(I_2 + I_1) + V_2 = 0$$
$$\boxed{V_2 = 6 I_1 + 9 I_2} \longrightarrow (4)$$

Comparing (1) & (3):

$$\boxed{Z_{11} = 18\,\Omega} \; ; \; \boxed{Z_{12} = 6\,\Omega}$$

Comparing (2) & (4):

$$\boxed{Z_{21} = 6\,\Omega} \; ; \; \boxed{Z_{22} = 9\,\Omega}$$

NOTE: 3 types of network:

1) T → preferred for Z
2) Π → preferred for Y
3) Bridge



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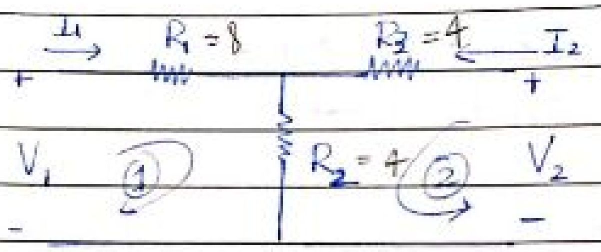
CONSTRUCTING NETWORK

(9)

* 2) Given: $Z = \begin{bmatrix} 12 & 4 \\ 4 & 8 \end{bmatrix}$ Find the elements of the network.

S: Standard Z parameter equations are:

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$$




$Z_{11} = 12\,\Omega$ , $Z_{12} = 4\,\Omega$
$Z_{21} = 4\,\Omega$ , $Z_{22} = 8\,\Omega$

KVL to loop (1):

$$\boxed{V_1 = I_1 (R_1 + R_2) + I_2 R_2} \longrightarrow (3)$$

loop (2): $\boxed{V_2 = I_1 R_2 + I_2 (R_2 + R_3)} \longrightarrow (4)$

Comparing (1) & (3)

$R_2 = Z_{12} \Rightarrow \boxed{R_2 = 4\,\Omega}$

$R_1 + R_2 = 12 \Rightarrow \boxed{R_1 = 8\,\Omega}$

$R_2 + R_3 = 8 \Rightarrow \boxed{R_3 = 4\,\Omega}$



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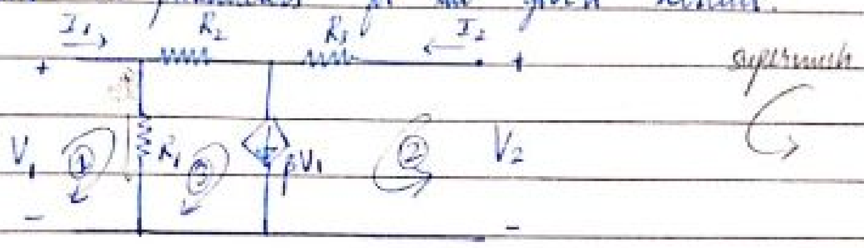
(10)

30-10-19

1) Find Z-parameters for the given circuit.




S: Standard Z-parameter equations are:

$$\boxed{V_1 = Z_{11} I_1 + Z_{12} I_2} \longrightarrow ①$$

$$\boxed{V_2 = Z_{21} I_1 + Z_{22} I_2} \longrightarrow ②$$

Applying KVL to loop ①:

$$-R_1(I_1 - I_3) V_1 = 0$$

$$\cancel{\boxed{R_1 I_1 = V_1}}$$

$$-R_1 I_1 + R_1(\beta V_1 - I_2) = -V_1$$

$$R_1 I_1 - \beta R_1 V_1 + R_1 I_2 = V_1$$

$$\boxed{R_1 I_1 + R_1 I_2 = V_1 (1 + \beta R_1)}$$

$$\boxed{V_1 = \frac{R_1 I_1}{1+\beta R_1} + \frac{R_1 I_2}{1+\beta R_1}} \longrightarrow ④$$

Constraint eq:

$$I_3 + I_2 = \beta V_1$$

$$\boxed{I_3 = \beta V_1 - I_2} \longrightarrow ③$$

Applying KVL to super mesh:

$$-R_2(I_3) - R_3 I_2 + V_2 - R_1(-I_3 + I_1) = 0$$

$$-R_1 I_1 - R_3 I_2 + I_3(R_2 + R_1) = -V_2$$

$$-R_1 I_1 - R_3 I_2 + (\beta V_1 - I_2)(R_2 + R_1) = -V_2$$

$$-R_1 I_1 - R_3 I_2 + \beta R_2 V_1 + \beta R_1 V_1 - I_2 R_2 - I_2 R_1 = -V_2$$

$$-R_1 I_1 - I_2(R_1 + R_2 + R_3) + V_1(\beta R_2 + \beta R_1) = -V_2$$

$$-R_1 I_1 - I_2(R_1 + R_2 + R_3) + \left(\frac{R_1 I_1}{1+\beta R_1} + \frac{R_1 I_2}{1+\beta R_1}\right)(\beta R_2 + \beta R_1) = -V_2$$

$$-R_1 I_1 - I_2(R_1 + R_2 + R_3) + \frac{\beta R_1 I_1 R_2}{1+\beta R_1} + \frac{\beta R_1^2 I_1}{1+\beta R_1} + \frac{\beta R_1 R_2 I_2}{1+\beta R_1} + \frac{\beta R_1^2 I_2}{1+\beta R_1} = V_2$$

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$$I_1\left[\frac{-R_1+\beta R_1R_2+\beta R_1^2}{1+\beta R_1}\right]+I_2\left[R_1+R_2+R_3+\frac{\beta R_1R_2+\beta R_1^2}{1+\beta R_1}\right]=-V_2$$

$$\frac{I_1R_1}{1+\beta R_1}\left[-1-\cancel{\beta R_1}+\cancel{\beta R_1}+\beta R_2\right]+I_2\left[-R_1-R_2-R_3+\frac{\beta R_1(R_2+R_1)}{1+\beta R_1}\right]=-V_2$$

$$-V_2=\frac{I_1R_1}{1+\beta R_1}\left(\beta R_2-1\right)+I_2\left[\frac{-R_1-\cancel{R_1^2\beta}-R_2-\beta R_2R_1-R_3(1+\beta R_1)+\cancel{\beta R_1 R_2 + \beta R_1^2}}{(1+\beta R_1)}\right]$$

$$I_1\,\frac{R_1\left[\beta R_2-1\right]}{1+\beta R_1}+I_2\left[-R_3-\frac{(R_1+R_2)}{(1+\beta R_1)}\right]=-V_2$$

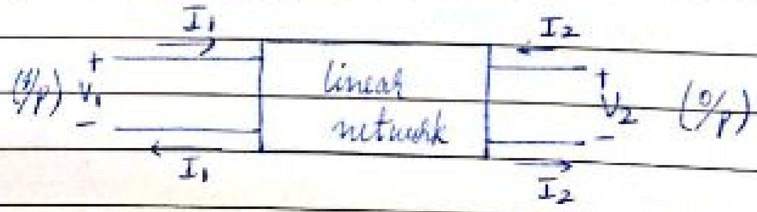
Cancelling (−)

$$\Rightarrow\ \boxed{I_1\,\frac{R_1(1-\beta R_2)}{1+\beta R_1}+I_2\left[R_3+\frac{(R_1+R_2)}{1+\beta R_1}\right]=V_2}$$

(2) $\Rightarrow$ $\boxed{Z_{11}=\frac{R_1}{1+\beta R_1}\,\Omega}$ $\quad$ $\boxed{Z_{12}=\frac{R_1}{1+\beta R_1}}\,\Omega$

$\boxed{Z_{21}=\frac{R_1(1-\beta R_2)}{1+\beta R_1}}\,\Omega$ $\quad$ $\boxed{Z_{22}=R_3+\frac{(R_1+R_2)}{1+\beta R_1}\,\Omega}$

## 2 • HYBRID PARAMETERS:




$V_1=h_{11}I_1+h_{12}V_2$

$I_2=h_{21}I_1+h_{22}V_2$

$h_{11}=\left.\frac{V_1}{I_1}\right|_{V_2=0}$ $\rightarrow$ Input impedance when o/p is short circuit.

$h_{12}=\left.\frac{V_1}{V_2}\right|_{I_1=0}$ $\rightarrow$ Reverse voltage gain when i/p open circuit.

$h_{21}=\left.\frac{I_2}{I_1}\right|_{V_2=0}$ $\rightarrow$ Forward current gain when o/p is short circuit



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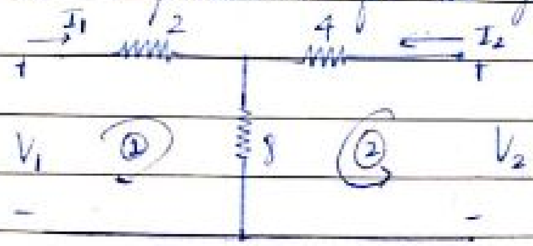
(12)

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$ → Output admittance when input is open circuit.

1) Find the h-parameters for the given circuit.




S: The standard h-parameter equations are:

$$\boxed{V_1 = h_{11} I_1 + h_{12} V_2} \longrightarrow (1)$$

$$\boxed{I_2 = h_{21} I_1 + h_{22} V_2} \longrightarrow (2)$$

Applying KVL to loop (1):

$$-2I_1 - 8(I_1 + I_2) + V_1 = 0$$

$$-10I_1 - 8I_2 = -V_1$$

$$\boxed{10I_1 + 8I_2 = V_1} \longrightarrow (3)$$

Loop (2):

$$-4I_2 - 8(I_2 + I_1) + V_2 = 0$$

$$\boxed{8I_1 + 12I_2 = V_2} \longrightarrow (4)$$

Rearranging eq. (4) ⇒ $\boxed{I_2 = -\frac{8}{12} I_1 + \frac{1}{12} V_2} \longrightarrow (5)$

Comparing (2) & (5)

$$\boxed{h_{21} = \frac{-2}{3}}, \quad \boxed{h_{22} = \frac{1}{12}\ \mho}$$

Substitute (5) in (3)

$$10I_1 + 8\left(\frac{-8}{12} I_1 + \frac{1}{12} V_2\right) = V_1$$

$$I_1\left(\frac{-64}{12} + 10\right) + \frac{8}{12} V_2 = V_1$$

$$\boxed{I_1\left(\frac{-56}{12}\right) + \frac{2}{3} V_2 = V_1} \longrightarrow (6)$$

** → Comparing (1) and (6), $\boxed{h_{11} = \frac{-14}{3}\,\Omega}$ and $\boxed{h_{12} = \frac{2}{3}}$



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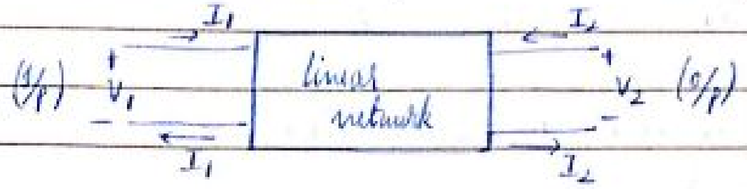
- **TRANSMISSION PARAMETERS** [Also called A,B,C,D parameters], T parameters, Transformer parameters.




$$\boxed{V_1 = AV_2 - BI_2}$$
$$\boxed{I_1 = CV_2 - DI_2}$$

(or)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ ⟶ Reverse voltage gain when o/p is O.C.

$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$ ⟶ Reverse transfer impedence when o/p is S.C.

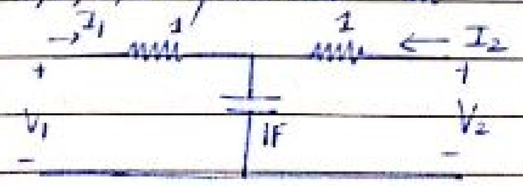
$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ ⟶ Reverse transfer admittence when o/p is O.C.

$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$ ⟶ Reverse current gain when o/p is S.C.

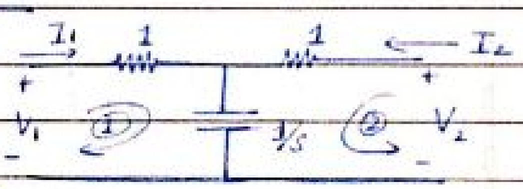
NOTE:

* 1) When a two port network doesn't contain any dependent source, then $\boxed{AD - BC = 1}$.

1) Determine A, B, C, D parameters in S-domain.




S:




→ The circuit in S-domain is

→ Standard equations:

$$\boxed{V_1 = AV_2 - BI_2} \longrightarrow ①$$
$$\boxed{I_1 = CV_2 - DI_2} \longrightarrow ②$$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(14)

Applying KVL to loop (1):

$$-I_1 - \frac{1}{S}[I_1+I_2] + V_1 = 0$$

$$\boxed{I_1\left[1+\frac{1}{S}\right] + \frac{1}{S}I_2 = V_1} \rightarrow (3)$$

loop (2):

$$-I_2 - \frac{1}{S}[I_2+I_1] + V_2 = 0$$

$$\frac{1}{S}I_1 + \left[1+\frac{1}{S}\right]I_2 = V_2$$

Rearranging,

$$I_1 = S\left[V_2 - \left(1+\frac{1}{S}\right)I_2\right]$$

$$\boxed{I_1 = SV_2 - (S+1)I_2} \rightarrow (4)$$

→ Comparing (2) & (4)

$$\boxed{C = S\,\mho} \quad ; \quad \boxed{D = (S+1)}$$

Substitute (4) in (3):

$$[SV_2 - (S+1)I_2]\left(1+\frac{1}{S}\right) + \frac{1}{S}I_2 = V_1$$

$$SV_2 + V_2 - (S+1)I_2 - \left(\frac{1+1}{S}\right)I_2 + \frac{1}{S}I_2 = V_1$$

$$I_2\left(-(S+1) - \left(\frac{1+1}{S}\right) + \frac{1}{S}\right) + (S+1)V_2 = V_1 \Rightarrow -I_2(S+2) + (S+1)V_2 = V_1$$

$$\boxed{(S+1)V_2 - I_2\left(\frac{\cancel{2}S^2 + 2\cancel{S+1}}{\cancel{S}}\right) = V_1} \rightarrow (5)$$

Comparing (1) and (5),

$$\boxed{A = S+1} \quad ; \quad \boxed{\cancel{B = \frac{2s^2+2s+1}{S}\,\Omega}}$$ check

$$\therefore \boxed{B = (S+2)\,\Omega}$$



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- **RELATIONSHIPS:**

1) Relationship between 'Z' & 'Y' parameters:

(a) 'Y' in terms 'Z' parameters.

W.k.t

$$\boxed{I_1 = Y_{11}V_1 + Y_{12}V_2} \longrightarrow (1)$$

$$\boxed{I_2 = Y_{21}V_1 + Y_{22}V_2} \longrightarrow (2)$$

$$\boxed{V_1 = Z_{11}I_1 + Z_{12}I_2} \longrightarrow (3)$$

$$\boxed{V_2 = Z_{21}I_1 + Z_{22}I_2} \longrightarrow (4)$$

$$(3) \Rightarrow \quad I_1 = \frac{V_1 - Z_{12}I_2}{Z_{11}} \Rightarrow \boxed{I_1 = \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}} I_2} \longrightarrow (5)$$

Subs (5) in eq. (4)

$$\boxed{V_2 = Z_{21}\left(\frac{V_1}{Z_{11}} - \frac{Z_{12}I_2}{Z_{11}}\right) \cancel{I_2} + Z_{22}I_2}$$

$$\Rightarrow V_2 = \frac{Z_{21}V_1}{Z_{11}} + I_2\left(Z_{22} - \frac{Z_{21}Z_{12}}{Z_{11}}\right)$$

$$I_2 = \frac{\left(V_2 - \frac{Z_{21}V_1}{Z_{11}}\right) Z_{11}}{(Z_{11}Z_{22} - Z_{21}Z_{12})} = \frac{Z_{11}V_2}{(Z_{11}Z_{22} - Z_{21}Z_{12})} - \frac{Z_{21}V_1}{(Z_{11}Z_{22} - Z_{12}Z_{21})}$$

$$\boxed{I_2 = \frac{V_1 Z_{21}}{(Z_{12}Z_{21} - Z_{11}Z_{22})} + \frac{(-)V_2 Z_{11}}{(Z_{12}Z_{21} - Z_{11}Z_{22})}}$$

Let $\Delta Z = Z_{11}Z_{22} - Z_{21}Z_{12}$

$$\Rightarrow \boxed{I_2 = \frac{Z_{21}}{-\Delta Z} V_1 + \frac{(-)Z_{11}}{\cancel{-}\Delta Z} V_2} \longrightarrow (6)$$

Using eq. (6) in (5)

$$I_1 = \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}}\left(\frac{-Z_{21}}{\Delta Z} V_1 + \frac{Z_{11}}{\Delta Z} V_2\right)$$



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$$I_1 = \frac{V_1}{Z_{11}} + \frac{Z_{21} Z_{12} V_1}{Z_{11} \Delta Z} - \frac{Z_{12}}{\Delta Z} V_2$$

$$\boxed{I_1 = \left(\frac{1}{Z_{11}} + \frac{Z_{21} Z_{12}}{Z_{11} \Delta Z}\right) V_1 - \frac{Z_{12}}{\Delta Z} V_2}$$

$$I_1 = \left(\frac{\cancel{-Z_{21} Z_{12}} + Z_{11} Z_{22} + \cancel{Z_{21} Z_{12}}}{\cancel{Z_{11}} \Delta Z}\right) V_1 - \frac{Z_{12}}{\Delta Z} V_2$$

$$\therefore \boxed{I_1 = \frac{Z_{22}}{\Delta Z} V_1 - \frac{Z_{12}}{\Delta Z} V_2} \longrightarrow (7)$$

31-10-14

- Relationship between 'Z' and 'T' parameters.

(A) 'Z' in terms of 'T'.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1) \qquad V_1 = AV_2 - BI_2 \longrightarrow (3)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2) \qquad I_1 = CV_2 - DI_2 \longrightarrow (4)$$

Rearranging eq. (4).

$$V_2 = \frac{I_1 + DI_2}{C}$$

$$\boxed{V_2 = \frac{I_1}{C} + \frac{D}{C} I_2} \longrightarrow (5)$$

$$\rightarrow \boxed{Z_{21} = \frac{1}{C}} \; ; \; \boxed{Z_{22} = \frac{D}{C}}$$

Substitute (5) in (3):

$$V_1 = A\left(\frac{I_1}{C} + \frac{D I_2}{C}\right) - BI_2$$

$$\boxed{V_1 = \left(\frac{A}{C} \cancel{- B}\right) I_1 + \left(\frac{AD}{C} - B\right) I_2} \longrightarrow (6)$$

$$\rightarrow \boxed{Z_{11} = \frac{A}{C}} \; ; \; \boxed{Z_{12} = \frac{AD - B}{C}}$$

(17)

(iv) $Z_{12} = \dfrac{AD - BC}{C}$

$$\boxed{Z_{12} = \frac{\Delta T}{C}} \quad \text{where } \boxed{\Delta T = AD - BC}.$$

(B) 'T' in Terms of Z

| T - parameters | Z - parameters |
| --- | --- |
| $V_1 = AV_2 - BI_2 \longrightarrow (1)$ | $V_1 = Z_{11}I_1 + Z_{12}I_2 \longrightarrow (3)$ |
| $I_1 = CV_2 - DI_2 \longrightarrow (2)$ | $V_2 = Z_{21}I_1 + Z_{22}I_2 \longrightarrow (4)$ |

Rearranging (4).
$$\boxed{I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22}I_2}{Z_{21}}} \longrightarrow (5)$$

Compare (2) & (5)
$$\boxed{C = \frac{1}{Z_{21}}} \; ; \; \boxed{D = \frac{Z_{22}}{Z_{21}}}$$

Substitute (5) in (3)
$$V_1 = Z_{11}\left(\frac{V_2}{Z_{21}} - \frac{Z_{22}I_2}{Z_{21}}\right) + Z_{12}I_2$$

$$\boxed{V_1 = \frac{Z_{11}}{Z_{21}}V_2 - \left(\frac{Z_{22}Z_{11} - Z_{12}}{Z_{21}}\right)I_2} \longrightarrow (6)$$

Comparing (1) & (6)
$$\therefore \boxed{A = \frac{Z_{11}}{Z_{21}}} \qquad \boxed{B = \left(\frac{Z_{11}Z_{22}}{Z_{21}} - Z_{12}\right)\Omega}$$



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STANDARD PROBLEMS

*1) Complete the given table and also find the values of Y-parameters.

| | $V_1$ | $V_2$ | $I_1$ | $I_2$ |
|---|---|---|---|---|
| 1 | 50 | 100 | -1 | 27 |
| 2 | 100 | 50 | 7 | 24 |
| 3 | 200 | 0 | 20 | 28 |
| 4 | 140.845 | -98.592 | 20 | 0 |
| 5 | 133.80 | 56.34 | 10 | 30 |


S: Y-parameter equations (in matrix form)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

2 trials
$$\begin{array}{c} I_1 \rightarrow \\ I_2 \rightarrow \end{array} \begin{bmatrix} -1 & 7 \\ 27 & 24 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} 50 & 100 \\ 100 & 50 \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 27 & 24 \end{bmatrix} \begin{bmatrix} 50 & 100 \\ 100 & 50 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 27 & 24 \end{bmatrix} \begin{bmatrix} -0.0067 & 0.0133 \\ 0.0133 & -0.0067 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix}} \; \mho$$

Calc: Steps → Mat A, Shift 4, Dim, Mat B, 2×2, Values, AC,
(Shift 4, Mat A) × (Shift 4, Mat B, Inverse)
= Ans



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$$A = BC$$
$$B^{-1}A = \underbrace{B^{-1}B}_{I}C$$

3) →

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 = 200 \\ V_2 = 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix} \begin{bmatrix} 200 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 28 \end{bmatrix}}$$

4) →

$$\begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 140.845 \\ -98.592 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}$$

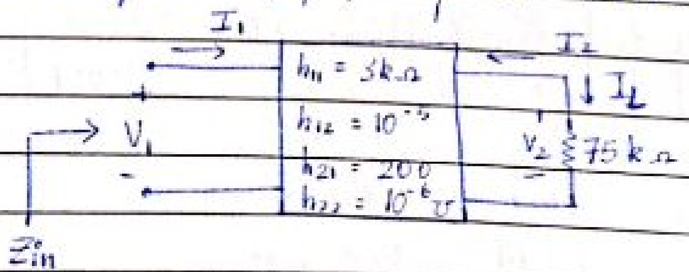
5) →

$$\begin{bmatrix} 10 \\ 30 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 30 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 133.80 \\ 56.34 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}$$

2) Find the impedence (input) of the network.




S: Standard equations of h-parameters,

$$\boxed{\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}}$$
$\rightarrow$ ① $\rightarrow$ ②



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$V_2 = I_L \times Z_L \Rightarrow \boxed{V_2 = -I_2 Z_L} \longrightarrow (3)$

→ Using this in eq. (2):

$I_2 = h_{21} I_1 + h_{22}(I_2 Z_L)$

$I_2 = h_{21} I_1 - h_{22} Z_L I_2$

$I_2(1 + h_{22} Z_L) = h_{21} I_1$

$$\boxed{I_2 = \frac{h_{21} I_1}{1 + h_{22} Z_L}} = \frac{200 I_1}{1 + (10^{-6} \times 75 \times 10^3)}$$

$$I_2 = \frac{200}{(1 + 75 \times 10^{-3})} I_1$$

$\boxed{I_2 = 186.046 I_1} \longrightarrow (4)$

→ Subs. (4) in (3)

$V_2 = -186.046 I_1 \times 75 \times 10^3$

$\boxed{V_2 = -13.95 \times 10^6 I_1} \longrightarrow (5)$

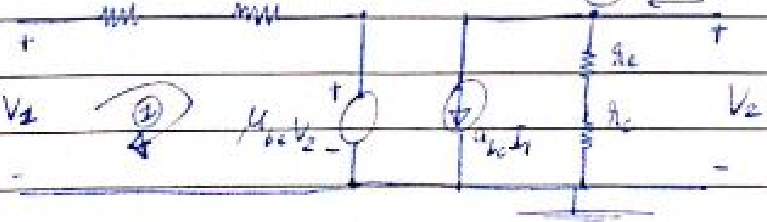
→ Substitute (5) in (1)

$V_1 = (3 \times 10^3) I_1 + (10^{-5} \times (-13.95 \times 10^6)) I_1$

$V_1 = I_1 (3 \times 10^3 - 13.95 \times 10^1)$

$\frac{V_1}{I_1} = \boxed{Z_{in} = 2.86\ k\Omega}$

* 3) The model of a transistor in CE mode is as shown in figure. Determine its 'h' parameters.



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(21)

Standard equations of 'h' parameter.

$$\boxed{V_1 = h_{11} I_1 + h_{12} V_2} \longrightarrow ①$$

$$\boxed{I_2 = h_{21} I_1 + h_{22} V_2} \longrightarrow ②$$

→ Applying KVL to loop ①:

$$-r_b I_1 - r_e I_1 - \mu_{bc} V_2 + V_1 = 0$$

$$\boxed{(r_b + r_e) I_1 + \mu_{bc} V_2 = V_1} \longrightarrow ③$$

Compare ① & ③:

$$\boxed{h_{11} = r_b + r_e}\ \Omega$$

$$\boxed{h_{12} = \mu_{bc}}$$

→ Applying KCL @ node ②:

$$\alpha_{bc} I_1 + \frac{V_2}{(r_e + r_c)} - I_2 = 0$$

$$\boxed{I_2 = \alpha_{bc} I_1 + \frac{V_2}{(r_e + r_c)}} \longrightarrow ④$$

Compare eq. ② & ④

$$\boxed{h_{21} = \alpha_{bc}}$$

$$\boxed{h_{22} = \frac{1}{r_e + r_c}\ \mho}$$



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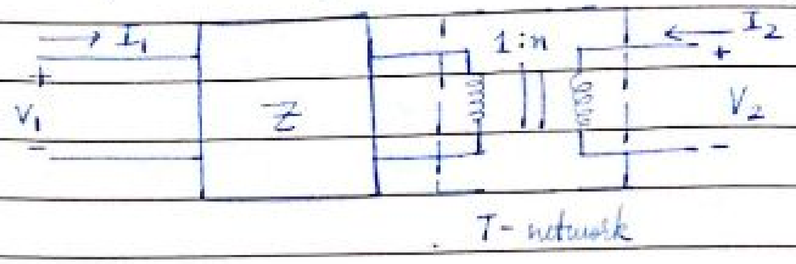
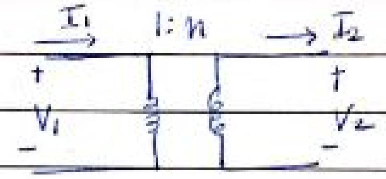
5) Find the Z-parameters of the overall network




S: Overall,

$$[T] = [T]_1 \times [T]_2$$

First, we should find out the transmission parameters to the network shown:




$$\frac{V_1}{V_2} = \frac{1}{n} = \frac{I_2}{I_1}$$

We know that,

$$\boxed{V_1 = AV_2 - BI_2} \rightarrow ①$$
$$\boxed{I_1 = CV_2 - DI_2} \rightarrow ②$$

For the given transformer in the circuit, we have,

$$\boxed{\frac{V_1}{V_2} = \frac{1}{n} = \frac{-I_2}{I_1}} \rightarrow ③$$

Let $I_2 = 0$ in ③:

$$\frac{V_1}{V_2} = \frac{1}{n}$$

$$\boxed{V_1 = \frac{1}{n} V_2} \rightarrow ④$$

Comparing ① and ④

$$\boxed{A = \frac{1}{n}} \quad ; \quad \boxed{B = 0}$$

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Similarly,

$$\frac{-I_2}{I_1} = \frac{1}{n}$$

$$\boxed{I_1 = -n I_2} \rightarrow ⑤$$

Comparing ② & ⑤:

$$\boxed{C=0} \; ; \; \boxed{D = n}.$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/n & 0 \\ 0 & n \end{bmatrix}} \longrightarrow ⑥$$

→ The 'Z' parameters of the first part of the circuit are:

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

→ 'T' in terms of 'Z', we have:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} Z_{11}/Z_{21} & \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} \\ 1/Z_{21} & Z_{22}/Z_{21} \end{bmatrix} \longrightarrow ⑦$$

The overall A, B, C, D parameters of the network can be obtained by multiplying the equations ⑤ & ⑥.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} Z_{11}/Z_{21} & \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} \\ 1/Z_{21} & Z_{22}/Z_{21} \end{bmatrix} \begin{bmatrix} 1/n & 0 \\ 0 & n \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_T = \begin{bmatrix} \frac{Z_{11}}{Z_{21} \times n} + 0 & \frac{n(Z_{11}Z_{22} - Z_{21}Z_{12})}{Z_{21}} \\ \frac{1}{n Z_{21}} & n \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

(overall A, B, C, D)



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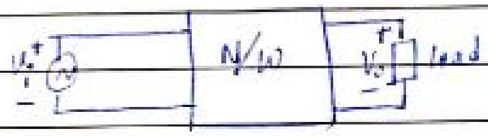
Overall 'Z',

$$[Z]_T = \begin{bmatrix} A/C & \Delta T/C \\ 1/C & D/C \end{bmatrix}$$

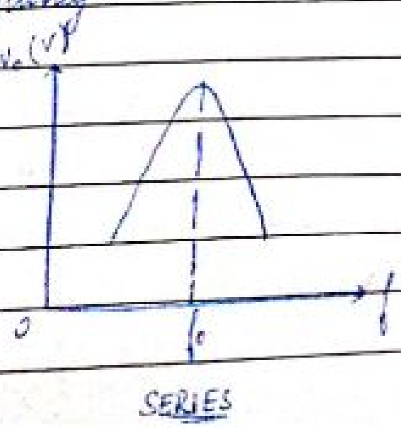
$$[Z]_T = \begin{bmatrix} Z_{11} & \dfrac{\frac{Z_{11}}{nZ_{21}} \times \frac{nZ_{22}}{Z_{21}} - \frac{Z_{11}Z_{22} + Z_{12}Z_{21}}{Z_{21}^2}}{1/nZ_{21}} \\ nZ_{21} & \frac{nZ_{22}}{Z_{21}} \times nZ_{21} \end{bmatrix}$$

$$[Z]_T = \begin{bmatrix} Z_{11} & nZ_{12} \\ nZ_{21} & n^2 Z_{22} \end{bmatrix}$$

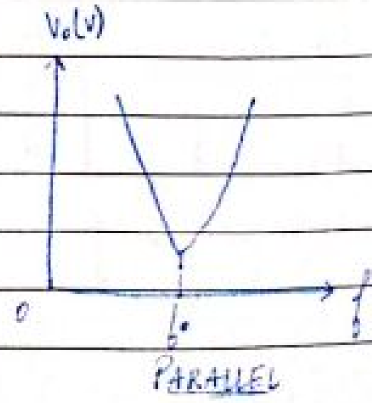
- **RESONANT CIRCUITS:**




A given network will resonate when the net reactance, i.e., X = 0. A two port network in general offers a complex impedance, consisting of resistive and reactive components.
At certain conditions, the impedance offered by the network is purely resistive ; such a phenomena is called RESONANCE. The frequency at which this happens is called resonant frequency.




SERIES



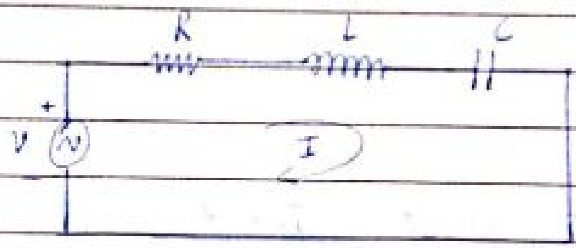

PARALLEL

* AC circuit becomes resonant circuit when the current drawn from the supply is in phase with the applied sinusoidal voltage. Then,
1) Resultant reactance = 0
2) Circuit behaves as a resistive circuit.
3) P.f. = 1 [Power factor]

- TYPES OF RESONANT CIRCUITS:

1) Series resonant circuit
2) Parallel resonant circuit [Anti-resonant]

- SERIES RESONANCE:




→ Consider the series RLC circuit shown above, where $V$ = sinusoidal voltage, $I$ = current flowing in the loop.
→ The circuit is said to be resonant when the resultant reactance of the circuit is zero.

→ The impedance of the given circuit at any frequency is:

$$\boxed{Z = R + j(X_L - X_C)} \longrightarrow (1)$$

$$Z = R + j\left[\omega L - \frac{1}{\omega C}\right] \longrightarrow (2)$$

$$\boxed{I = \frac{V}{Z} = \frac{V}{R + j(X_L - X_C)}} \longrightarrow (3)$$



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→ At resonance, the circuit must have unity P.F.

i.e,

$$P.F. = \cos\phi = \frac{R}{Z}$$

i.e, $X_L - X_C = 0$

$$\Rightarrow \omega L - \frac{1}{\omega C} = 0$$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

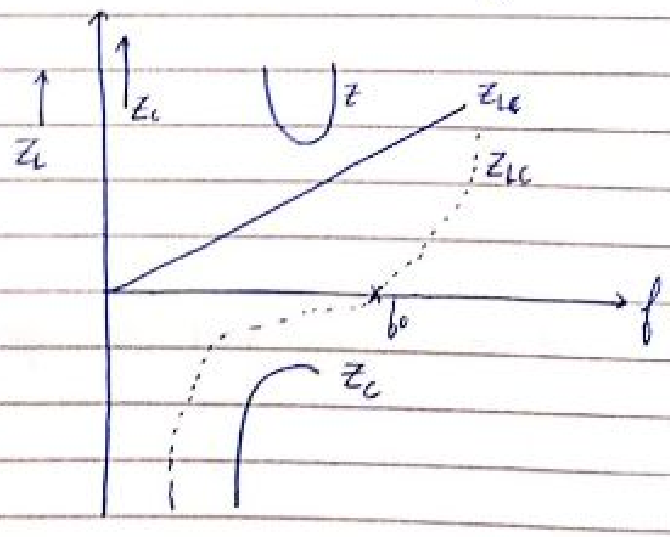
$$\Rightarrow \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz}$$

(or)

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \text{ rad/sec}$$

→ $f_0$ = Resonant frequency

7-11-19 → At resonance, the current, $I_0 = \frac{V}{R}$.




(1) → For $f < f_0$, $Z_{LC}$ becomes capacitive



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(2) For $f > f_0$,

$Z_{eq}$ becomes inductive.

$$\boxed{|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

- VARIATION OF CURRENT, VOLTAGE WITH FREQUENCY:

→ The impedance of series RLC circuit is

$$\boxed{Z = R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$I = \frac{V}{Z} \Rightarrow \boxed{I = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}}$$

✓** • Expression for frequency at which $V_C$ (Voltage across capacitor) is maximum:

$$V_C = \frac{I}{j\omega C}$$

$$\Rightarrow V_C = \frac{V}{\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right] \times j\omega C}$$

$$\boxed{|V_C| = \frac{V}{\omega C\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}} \longrightarrow (1)$$

Squaring on both sides,

$$V_C^2 = \frac{V^2}{(\omega C)^2\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)}$$

→ The frequency at which $V_C$ is maximum is obtained by

$$\frac{\partial V_C^2}{\partial \omega} = 0$$

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$$\Rightarrow \frac{\partial V_c^2}{\partial \omega} = 0 = \frac{V^2}{C^2}\frac{\partial}{\partial \omega}\left[\frac{1}{\omega^2\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]}\right]$$

$$0 = \frac{V^2}{C^2}\left[\frac{0-\frac{\partial}{\partial \omega}\left[\omega^2\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]\right]}{\left[\omega^2\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]\right]^2}\right]$$

$$\Rightarrow \frac{\partial}{\partial \omega}\left[\omega^2R^2+\left(\omega^2L-\frac{1}{C}\right)^2\right] = 0$$

$$2R^2\omega + 2\left(\omega^2L-\frac{1}{C}\right)(2\omega L) = 0$$

$$\cancel{2R^2\omega +} \underbrace{2\omega}_{\text{can't be zero}}\left[R^2+2L\left(\omega^2L-\frac{1}{C}\right)\right] = 0$$

$$\Rightarrow R^2 + 2L\left(\omega^2L-\frac{1}{C}\right) = 0$$

$$R^2 + 2\omega^2L^2 - \frac{2L}{C} = 0$$

$$R^2C + 2\omega^2L^2C \cancel{+} - 2L = 0$$

$$2\omega^2L^2C = 2L - R^2C$$

$$\omega^2 = \frac{2L-R^2C}{2L^2C}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\boxed{\omega = \sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}}$$

$$\omega = \frac{1}{\sqrt{LC}}\left(\sqrt{1-\frac{R^2C}{2L}}\right)$$

(For $V_{c(max)}$) $\boxed{\omega_c = \omega_0\left(\sqrt{1-\frac{R^2C}{2L}}\right)}$ rad/sec

$$\Rightarrow \boxed{f_c = f_0\sqrt{1-\frac{R^2C}{2L}}\ \text{Hz}}$$

($f_c$ is frequency at $V_{c(max)}$)



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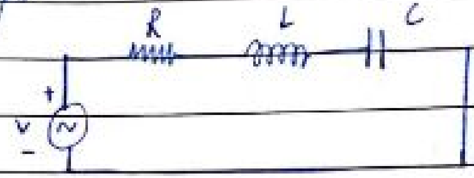
1) A series RLC circuit has $R = 25\,\Omega$, $L = 0.04\,H$ & $C = 0.01\,\mu F$. Calculate the resonant frequency and also if a 1V source of same frequency as the resonant frequency is applied to the circuit. Calculate the frequencies at which the voltage across 'L' and 'C' are maximum and their respective voltage. [Circuit not given].

S:




→ $$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.04 \times 0.01 \times 10^{-6}}}$$

$$\boxed{f_0 = 7957.74\,Hz}$$

→ $$f_C = f_0\sqrt{1 - \frac{R^2C}{2L}} = 7957.74\sqrt{1 - \frac{(25)^2 \times 0.01 \times 10^{-6}}{2 \times 0.04}}$$

$$\boxed{f_C = 7957.342\,Hz}$$

→ $$f_L = \frac{f_0}{\sqrt{1 - \frac{R^2C}{2L}}} = \frac{f_0}{0.9999} = \frac{7957.74}{0.9999}$$

$$\boxed{f_L = 7958.05\,Hz}$$

→ $V_L = I.X_L$ [at Resonance]

$\Rightarrow V_L = I_0 W_0 L$

$$V_L = \frac{V}{R} W_0 L = \frac{1}{25} \times 2\pi f_0 \times 0.04$$

$$V_L = \frac{0.04 \times 2\pi \times 7957.74}{25}$$

$$\boxed{V_L = 79.9999\,V}$$



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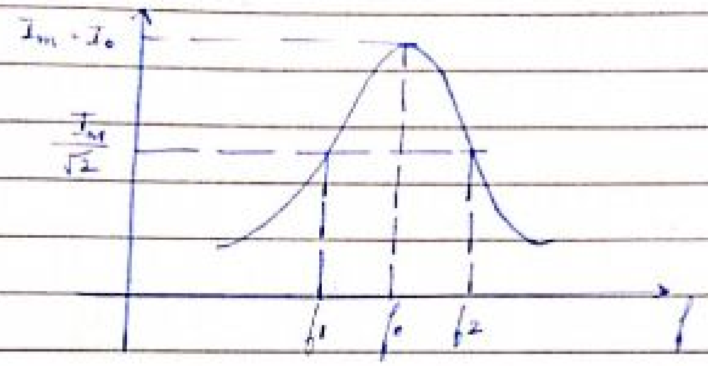
→ $V_c = IX_c = I_0 \times \frac{1}{\omega_0 C} = \frac{V}{R} \times \frac{1}{\omega_0 C}$

$$V_c = \frac{1}{25} \times \frac{1}{2\pi \times 7957.74 \times 0.01 \times 10^{-6}}$$

$$\boxed{V_c = 80 \ V}$$

• SELECTIVITY AND BANDWIDTH:




→ At resonant frequency, the impedance of RLC circuit is minimum. Hence, the current is maximum.

→ As the frequency of the applied voltage deviates on either side of the resonant frequency, the impedance increases and the current decreases. The above figure shows the variation of current with frequency. The frequencies $f_1$ and $f_2$ are called half power frequencies or 3 Db frequencies.

→ The bandwidth, $(f_2 - f_1)$ is called half power bandwidth or 3 Db bandwidth.

→ SELECTIVITY: Selectivity of a resonant circuit is defined as the ratio of resonant frequency to the bandwidth.

$$\text{Selectivity} = \frac{f_0}{BW} = \frac{f_0}{f_2 - f_1}$$



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- To prove that the power at frequency $f_1$ and $f_2$ is half of the power at resonance.

→ The current in series RLC circuit is:

$$I = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \longrightarrow ①$$

At resonant frequency, $I = I_0 = \frac{V}{R} \longrightarrow ②$

At upper cut-off frequency ($f_2$)/upper half power frequency:

$$|I_2| = \frac{|I_0|}{\sqrt{2}} = \frac{|V|}{R\sqrt{2}} = \left|\frac{V}{R + \underbrace{jR}_{\text{Inductive}}}\right|$$

$$|I_2| = \left|\frac{V}{R + jR}\right| \longrightarrow ③.$$

From eq. ① @ $f_2$:

$$\boxed{I_2 = \frac{V}{R + j\left(\omega_2 L - \frac{1}{\omega_2 C}\right)}} \longrightarrow ④$$

Comparing eq. ③ & ④

$$\boxed{R = \omega_2 L - \frac{1}{\omega_2 C}}$$

Similarly, at lower cut-off frequency ($f_1$),

$$|I_1| = \frac{|I_0|}{\sqrt{2}} = \frac{|V|}{R\sqrt{2}} = \left|\frac{V}{R - jR}\right|$$

$$|I_1| = \left|\frac{V}{R - jR}\right| \longrightarrow ⑤$$

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From eq. (1) @ $f_1$:

$$I_1 = \frac{V}{R + j\left(\omega_1 L - \frac{1}{\omega_1 C}\right)} \longrightarrow (7)$$

Comparing equations (6) & (7):

$$\boxed{R = \frac{1}{\omega_1 C} - \omega_1 L} \longrightarrow (8)*$$

→ The current ratio is expressed in decibels as $20 \log_{10}\left(\frac{I_1}{I_0}\right)$

$$\Rightarrow 20 \log_{10}\left(\frac{\frac{I_0}{\sqrt{2}}}{I_0}\right)$$

$$\Rightarrow -3 \text{ dB}$$

→ The power dissipated at the resonant frequency,

$$\boxed{P_0 = I_0^2 R}$$

→ The power dissipated at lower cut-off frequency is

$$P_1 = I_1^2 R$$

$$P_1 = \frac{I_0^2 R}{2}$$

$$\boxed{P_1 = \frac{P_0}{2}}$$

→ Similarly, $\boxed{P_2 = \frac{P_0}{2}}$

Therefore, the power at the frequencies, $f_1$ and $f_2$ is half of the power at resonance. Hence, they are called as half power frequencies.



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## • Transfer Function:

A transfer function, $H(j\omega)$ is also known as network function, it is an important tool for finding the frequency response of a circuit.

A transfer function is defined as the ratio of output response $Y(j\omega)$ to the sinusoidal i/p signal $X(j\omega)$.

i.e., $$\boxed{H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}}$$

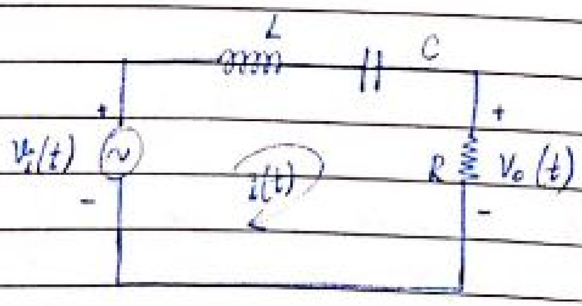
→ The roots of $Y(j\omega) = 0$ are called as zeros of $H(j\omega)$ and are represented as $j\omega = z_1, z_2, \ldots$

→ Similarly, the roots of $X(j\omega) = 0$ are called as Poles of $H(j\omega)$ and are represented as $j\omega = P_1, P_2, \ldots$

→ To avoid complexity, $(j\omega)$ will be replaced by 'S' while working and then 'S' is replaced back $(j\omega)$ at the end.

i.e, $$\boxed{H(S) = \frac{Y(S)}{X(S)}}$$

## • Transfer Function of Series Resonant Circuit:




$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

$\phi(j\omega) = 90 - \tan^{-1}\left(\frac{Y}{X}\right)$



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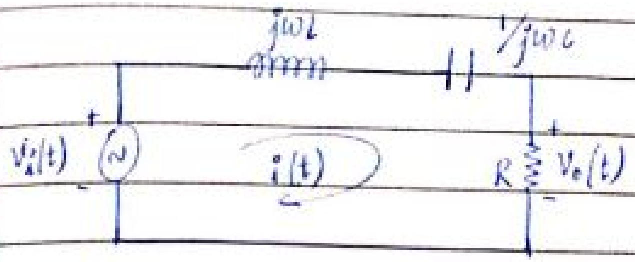
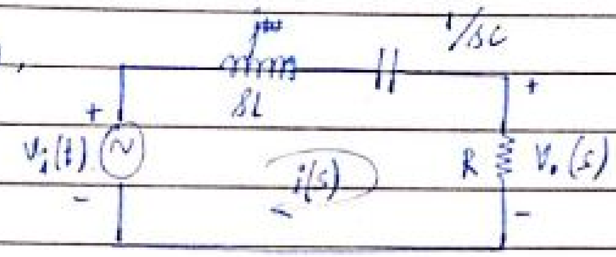
Consider the series resonant circuit as shown:




→ In 'S' domain,




From this circuit,

$$i(s) = \frac{V_i(s)}{R + \left(Ls + \frac{1}{sC}\right)}$$

→ $$V_o(s) = R\, i(s)$$

$$V_o(s) = \frac{V_i(s)\, R}{R + SL + \frac{1}{SC}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R}{R + SL + \frac{1}{SC}}$$

$$\Rightarrow H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + SL + \frac{1}{SC}}$$

$$= \frac{RSC}{RSC + LCS^2 + 1}$$

$$= \frac{RSC}{LC\left(S^2 + \left(\frac{R}{L}\right)S + \frac{1}{LC}\right)}$$

$$\boxed{H(s) = \frac{\left(\frac{R}{L}\right)s}{S^2 + \left(\frac{R}{L}\right)S + \frac{1}{LC}}}$$



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Substituting (s) by (jω)

$$\Rightarrow \boxed{H(j\omega) = \frac{\left(\frac{R}{L}\right) j\omega}{(j\omega)^2 + \left(\frac{R}{L}\right) j\omega + \frac{1}{LC}}} = \frac{\left(\frac{R}{L}\right) j\omega}{\left(\frac{1}{LC} - \omega^2\right) + \left(\frac{R}{L}\right) j\omega}$$

Taking the magnitude,

$$|H(j\omega)| = \frac{\left(\frac{R}{L}\right)\omega}{\sqrt{\underbrace{\left(\frac{1}{LC} - \omega^2\right)^2}_{X^2} + \underbrace{\left(\frac{\omega R}{L}\right)^2}_{Y^2}}} \longrightarrow (1)$$

$$\rightarrow \theta(j\omega) = \underline{/H(j\omega)} = 90 - \tan^{-1}\left(\frac{\frac{\omega R}{L}}{\frac{1}{LC} - \omega^2}\right) \rightarrow 90 - \tan^{-1}\left(\frac{Y}{X}\right)$$

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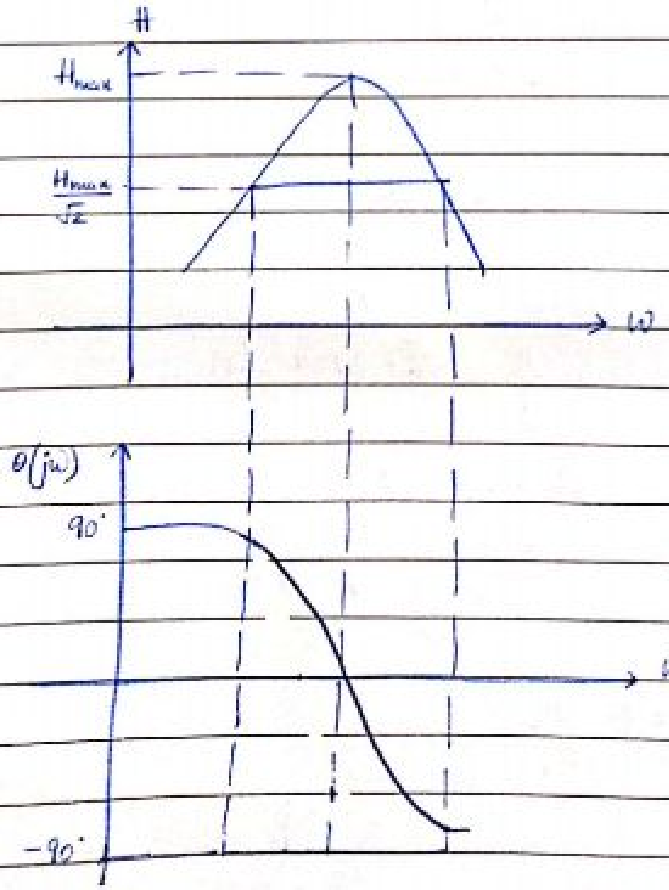




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$H_{max} = 1$

$$\boxed{H_{max} = |H(j\omega)|_{\omega=\omega_0}}$$

→ Taking $\omega = \omega_0$ in eq. ①  $\left[\omega = \omega_0 = \frac{1}{\sqrt{LC}}\right]$

$$\therefore |H(j\omega)| = \frac{\frac{R}{L} \times \frac{1}{\sqrt{LC}}}{\sqrt{\left(\frac{1}{LC} - \frac{1}{LC}\right)^2 + \left(\frac{1}{\sqrt{LC}} \times \frac{R}{L}\right)^2}}$$

$$= \frac{R \times 1}{L\sqrt{LC}} \times \frac{1}{\sqrt{0 + \frac{R}{L}\left(\frac{1}{\sqrt{LC}}\right)^2}}$$

$$= \frac{\cancel{R} \times \cancel{1}}{\cancel{L}\cancel{\sqrt{LC}}} \times \frac{1}{\frac{\cancel{R} \times 1}{\cancel{L}\sqrt{LC}}}$$

$$= 1$$

$$\therefore \boxed{H_{max} = 1}$$

- EXPRESSION FOR CUT-OFF FREQUENCY, BANDWIDTH AND QUALITY FACTOR.

let $(\omega_c)$ denote the angular cut-off frequency at ~~f~~ half power points in the frequency response.

We know that, at resonance,

$$I_0 = \frac{V}{R}$$

$$\boxed{\frac{I_0}{\sqrt{2}} = \frac{V_0}{R\sqrt{2}}} \rightarrow ①$$

We know that,

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



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$$\Rightarrow |Z| = \sqrt{R^2 + \left(\omega_c L - \frac{1}{\omega_c C}\right)^2}$$

But,

$$\frac{I_0}{\sqrt{2}} = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + \left(\omega_c L - \frac{1}{\omega_c C}\right)^2}} \longrightarrow (2)$$

Comparing eq. (1) and (2):

$$\sqrt{2} R = \sqrt{R^2 + \left(\omega_c L - \frac{1}{\omega_c C}\right)^2}$$

Squaring on both sides.

$$2R^2 = R^2 + \left(\omega_c L - \frac{1}{\omega_c C}\right)^2$$

$$R^2 = \left(\omega_c L - \frac{1}{\omega_c C}\right)^2$$

$$\cancel{\omega_c C R = \omega_c^2 L C - 1}$$

$$\cancel{\frac{\omega_c^2 L C - \omega_c C R - 1 = 0}{\omega_c}}$$

Taking square root,

$$\pm R = \left(\omega_c L - \frac{1}{\omega_c C}\right)$$

$$\omega_c^2 L - \frac{1}{C} = \pm \omega_c R$$

$$L\omega_c^2 \pm \omega_c R - \frac{1}{C} = 0$$

Thus, we have,

$$\omega_c = \frac{\pm R \pm \sqrt{R^2 + \frac{4L}{C}}}{2L}$$

$$\boxed{\omega_c = \frac{\pm R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$



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→ From the above equation, we can obtain '4' solutions, out of which only 2 are positive:

$$W_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \longrightarrow (3)$$

$$W_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \longrightarrow (4)$$

→ $$f_{c1} = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$f_{c2} = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

→ Usually, $\left(\frac{R}{2L}\right)$ will be a small value. Therefore, $\left(\frac{R}{2L}\right)^2$ will be further small. Hence, it may be neglected.

Thus,

$$f_{c1} = \frac{1}{2\pi}\left[\frac{-R}{2L} + \sqrt{\frac{1}{LC}}\right]$$

$$f_{c2} = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\frac{1}{LC}}\right]$$

→ $$B.W = f_{c2} - f_{c1} = \frac{1}{2\pi}\left[+\frac{R}{2L} + \cancel{\sqrt{\frac{1}{LC}}} + \frac{R}{2L} - \cancel{\sqrt{\frac{1}{LC}}}\right]$$

$$\boxed{B.W = \frac{R}{2\pi L}}$$

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→ Quality factor (Q),

$$Q = \frac{f_0}{BW} = \frac{2\pi f_0 L}{R}$$

$$\boxed{Q = \frac{\omega_0 L}{R}}$$

- RESONANT FREQUENCY IN TERMS OF HALF POWER FREQUENCIES:

Prove that the resonant frequency is the geometric mean of the two half power frequencies.

$$\boxed{f_0 = \sqrt{f_1 f_2}}$$

PROOF: We know that, @ $\omega_1$, $\boxed{R = \frac{1}{\omega_1 C} - \omega_1 L}$ ⟶ ①

Similarly, @ $\omega_2$, $\boxed{-R = \frac{1}{\omega_2 C} - \omega_2 L}$ ⟶ ②

$$R = \omega_2 L - \frac{1}{\omega_2 C} \longrightarrow ③$$

Equating ① & ③:

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = L(\omega_1 + \omega_2)$$

$$\frac{\cancel{(\omega_2 + \omega_1)}}{C\,\omega_1\omega_2} = L\cancel{(\omega_1 + \omega_2)}$$

$$\frac{1}{\omega_1\omega_2} = LC$$

$$\Rightarrow \boxed{\omega_1\omega_2 = \frac{1}{LC}}$$



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Taking square root, we have:

$$\sqrt{\omega_1 \omega_2} = \frac{1}{\sqrt{LC}} = \omega_0$$

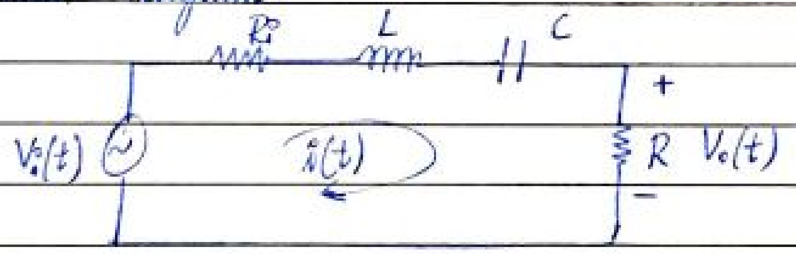
$$\sqrt{2\pi f_1 \times 2\pi f_2} = 2\pi f_0$$

$$\Rightarrow \cancel{2\pi} f_0 = \cancel{2\pi} \sqrt{f_1 f_2}$$

$$\therefore \boxed{f_0 = \sqrt{f_1 f_2}}$$

1) Determine the equation for transfer function for the given circuit diagram.

H.W.




S:

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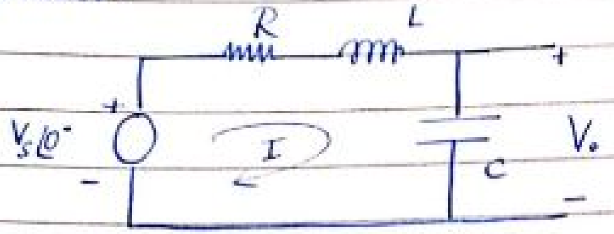
** DERIVATION

12-11-19 1) For the given circuit, show that $|V_o|_{max} = \frac{Q|V_s|}{\sqrt{1-\frac{1}{4Q^2}}}$, at resonance.

S:




i) $|V_o| = ?$ → ①

ii) $\frac{\partial |V_o|}{\partial \omega} = 0$ → $\omega_{max}$

iii) Subs. in eq. ①

$$V_o = I(-jX_c) \quad [\because V_o = 'V' \text{ across capacitor}]$$

$$V_o = \frac{V_s\angle 0^\circ}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \times \frac{-j}{\omega C} \rightarrow \frac{1}{\sqrt{(\omega c)^2}}$$

$$|V_o| = \frac{|V_s|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \times \frac{1}{\omega C}$$

$$\boxed{|V_o| = \frac{|V_s|}{\sqrt{(\omega^2 LC - 1)^2 + (\omega RC)^2}}} \rightarrow ①$$

$$|V_o| = \frac{|V_s|}{\sqrt{\underbrace{1 + \omega^4 L^2 C^2 - 2\omega^2 LC + \omega^2 R^2 C^2}_{X}}}$$

$$|V_o| = \frac{|V_s|}{\sqrt{X}}$$

Differentiate partially w.r.t '$\omega$':

$$\frac{\partial |V_o|}{\partial \omega} = |V_s|\left[\underbrace{\frac{-1}{2} X^{-3/2}}_{\text{this is constant} \Rightarrow \text{can't be zero.}} \frac{\partial X}{\partial \omega}\right] = 0$$

$$\frac{\partial X}{\partial \omega} = \frac{\partial}{\partial \omega}\left(1 + \omega^4 L^2 C^2 - 2\omega^2 LC + \omega^2 R^2 C^2\right)$$

$$= 4\omega^3 L^2 C^2 - 4\omega LC + 2\omega R^2 C^2$$

$$\Rightarrow 4\omega^3 L^2 C^2 - 4\omega LC + 2\omega R^2 C^2 = 0$$

$$2\omega C\left[2\omega^2 L^2 C - 2L + R^2 C\right] = 0$$

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$$2\omega^2L^2C - 2L + R^2C = 0$$

$$2\omega^2L^2C = 2L - R^2C$$

$$\omega^2 = \frac{2L - R^2C}{2L^2C} = \frac{1}{LC}\left[1 - \frac{R^2C}{2L}\right] \longrightarrow (2)$$

$$\omega^2 = \frac{1}{LC} - \frac{1}{2}\left(\frac{R}{L}\right)^2$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{1}{2}\left(\frac{R}{L}\right)^2}$$

We know that,

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\boxed{Q^2 = \frac{L}{R^2C}}$$

(1) $\Rightarrow$ $$\omega^2 = \frac{1}{LC}\left[1 - \frac{1}{2Q^2}\right]$$

$$\omega = \sqrt{\frac{1}{LC}\left(1 - \frac{1}{2Q^2}\right)} = \frac{1}{\sqrt{LC}}\sqrt{\left(1 - \frac{1}{2Q^2}\right)}$$

$$\omega = \omega_0\sqrt{\left(1 - \frac{1}{2Q^2}\right)}$$

$$\Rightarrow \boxed{\omega_{max} = \omega_0\sqrt{\left(1 - \frac{1}{2Q^2}\right)}} \longrightarrow (3)$$

↳ as $\left[\frac{\partial|V_0|}{\partial\omega} = 0\right.$ is condition for $\left.\omega_{max}\right]$

Substituting for $\omega_{max}$ in eq. (1), we have:

$$|V_0|_{max} = \frac{|V_s|}{\sqrt{(\omega_{max}^2LC - 1)^2 + (\omega_{max}RC)^2}} \longrightarrow (4)$$

$$\rightarrow (\omega_{max}RC)^2 = \left(\omega_0\sqrt{\left(1 - \frac{1}{2Q^2}\right)}RC\right)^2 = (\omega_0RC)^2 \times \left(1 - \frac{1}{2Q^2}\right)$$

$$= \frac{1}{LC} \times R^2C^{\cancel{2}}\left(1 - \frac{1}{2Q^2}\right) = \frac{R^2C}{L}\left(1 - \frac{1}{2Q^2}\right)$$



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$$\rightarrow (\omega_{max} RC)^2 = \frac{1}{Q^2}\left[1 - \frac{1}{2Q^2}\right]$$

$$= \boxed{\frac{1}{Q^2} - \frac{1}{2Q^4}}$$

$$\rightarrow (\omega_{max}^2 LC - 1)^2 = \left[\omega_0^2\left(1 - \frac{1}{2Q^2}\right)LC - 1\right]^2$$

$$= \left(\frac{-1}{2Q^2}\right)^2$$

$$= \boxed{\frac{1}{4Q^4}}$$

Substitute in eq. (4)

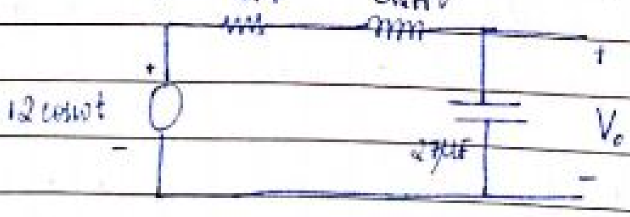
$$|V_0|_{max} = \frac{|V_s|}{\sqrt{\frac{1}{4Q^4} + \frac{1}{Q^2} - \frac{1}{2Q^4}}} = \frac{|V_s|}{\sqrt{\frac{1}{Q^2} - \frac{1}{4Q^4}}}$$

$$|V_0|_{max} = \frac{|V_s|}{\frac{1}{Q}\sqrt{1 - \frac{1}{4Q^2}}}$$

$$\boxed{|V_0|_{max} = \frac{Q\,|V_s|}{\sqrt{1 - \frac{1}{4Q^2}}}}$$ at resonance.

Hence, proved.

2) For the given circuit, find $\omega_0$, $Q_0$ & $|V_0|_{max}$.




S:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3\times10^{-3} \times 27\times10^{-6}}} = \frac{1}{0.9\times10^{-4}} = 3.513k \text{ rad/s}$$



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$$V_s = \frac{V_m}{\sqrt{2}} = V_{rms}, \quad V_m = 12$$

$$\cancel{Q_0 = \frac{L}{R^2 C} = \frac{3\times10^{-3}}{(0.4)^2 \times 27\times10^{-6}}}$$

$$V_s = \frac{12}{\sqrt{2}} = \underline{8.48}$$

$$Q_0 = \frac{W_0 L}{R} = \frac{3.513k \times 3mH}{0.4}$$

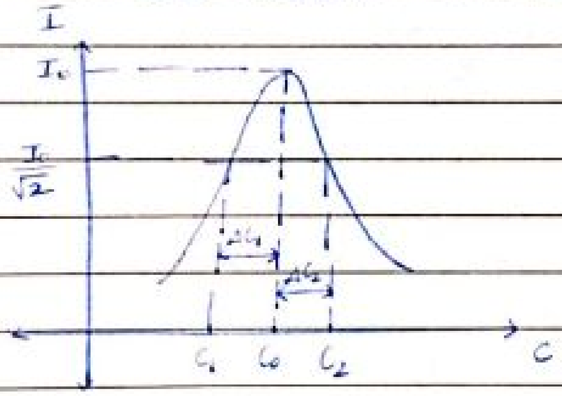
$$|V_c|_{max} = \frac{|V_s|\, Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

$$\boxed{Q_0 = 26.35}$$

$$|V_c|_{max} = \frac{8.48 \times 26.35}{\sqrt{1 - \frac{1}{4(26.35)^2}}} = \frac{223.448}{0.9998}$$

$$\boxed{|V_c|_{max} = 223.5\,V}$$

- SELECTIVITY ~~Q~~ WITH VARIABLE CAPACITANCE:




→ let $C_0$ be the capacitance at resonance.

→ $C_1$, $C_2$ are the value of capacitance at lower and upper cut-off frequencies respectively.

→ We know that,

$$W_0 L_1 - \frac{1}{W_0 C_1} = -R \quad \text{[from Geometric mean of f]}$$

$$WL_2 - \frac{1}{WC_2} = R$$

Here 'f' is constant. Thus it's not $W_0 L$, it's $WL_1$



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Here, $L_1 = L_2 = L$.

$$\Rightarrow \boxed{\omega L - \frac{1}{\omega C_1} = -R} \rightarrow (1)$$

$$\boxed{\omega L - \frac{1}{\omega C_2} = R} \rightarrow (2)$$

From equation (1) and (2), we have,

$$(2) - (1) \Rightarrow \frac{1}{\omega C_1} - \frac{1}{\omega C_2} = 2R$$

$$\boxed{\frac{C_2 - C_1}{\omega C_1 C_2} = 2R} \rightarrow (3)$$

But $C_2 = C_0 + \Delta C_2 \simeq C_0$

$C_1 = C_0 - \Delta C_1 \simeq C_0$

$\rightarrow \Rightarrow C_1 C_2 = C_0^2 \dashrightarrow$ only for denominator.

$$(3) \Rightarrow \frac{C_2 - C_1}{\omega C_0^2} = 2R$$

At resonance, net reactance $= 0$

$$\Rightarrow \frac{1}{\omega C_0} = \omega L$$

$$\Rightarrow \frac{C_2 - C_1}{C_0} = \frac{2R}{\omega L}$$

$$\frac{C_0}{C_2 - C_1} = \frac{\omega L}{2R}$$ [reciprocal]

$$\boxed{\frac{C_0}{C_2 - C_1} = \frac{Q}{2}}$$ $\left[\because \frac{\omega L}{R} = Q\right]$



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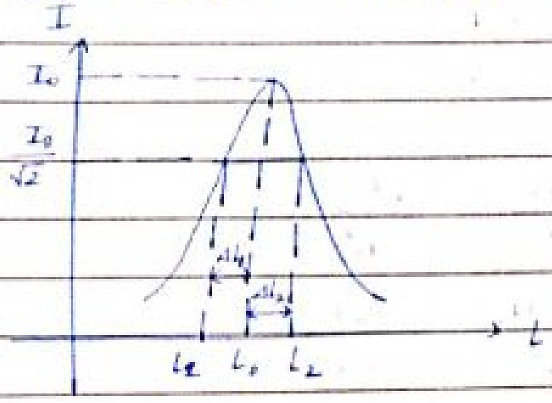
where $\frac{C_0}{C_2 - C_1}$ gives the selectivity of the series ~~for~~ resonant circuit with variable capacitance.

- SELECTIVITY WITH VARIABLE INDUCTANCE: [H.W.] $\frac{L_0}{L_2 - L_1} = \frac{Q}{2}$




→ Let $L_0$ be the impedence at resonance.

→

We know that,

$$\omega L_1 - \frac{1}{\omega C_1} = -R$$

$$\omega L_2 - \frac{1}{\omega C_2} = R$$

Here, ~~W~~ $C_1 = C_2 = C$

$$\Rightarrow \boxed{\omega L_1 - \frac{1}{\omega C} = -R} \longrightarrow (1)$$

$$\boxed{\omega L_2 - \frac{1}{\omega C} = R} \longrightarrow (2)$$

$$\frac{L_2 - L_1}{L_0} = \frac{2R}{\omega L_0}$$

(2) - (1) ⇒ $\omega L_2 - \omega L_1 = 2R$

$$L_2 - L_1 = \frac{2R}{\omega}$$

$$\frac{L_0}{L_2 - L_1} = \frac{\omega L_0}{2R}$$

$$\cancel{L_1 L_2 (L_2 - L_1) = \frac{2R(L_1 L_2)}{\omega}}$$

$$\boxed{\frac{L_0}{L_2 - L_1} = \frac{Q}{2}}$$

$\left\{\text{Since } Q = \frac{\omega L_0}{R}\right\}$

~~[mul by $(L_1 L_2)$ on both sides]~~

1) Series $Z = X + jY$; $Y = 0$, $\omega_0 = ?$, $f_0 = ?$

2) Parallel $Y = \alpha + j\beta$ [Admittance], $\beta = 0$, $\omega_0 = ?$, $f_0 = ?$

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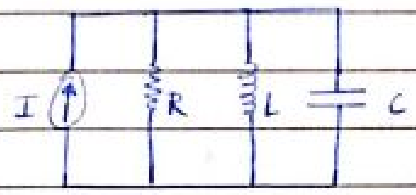
- **PARALLEL RESONANT CIRCUIT : [ANTI RESONANT CIRCUIT]**

TYPES: 1) Dual of series resonant circuit
2) General case RL - RC parallel circuit
3) Practical C-RL parallel resonant circuit.

## 1) DUAL OF SERIES RESONANT CIRCUIT :

"Duality" means opposite ⇒ [Current becomes voltage, & vice versa. Series becomes parallel & vice versa]




From the circuit, the net admittance is :

$$Y = Y_R + Y_L + Y_C = \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{1/j\omega C}$$

$$Y = \frac{1}{R} - \frac{j}{\omega L} + j\omega C$$

$$\boxed{Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

At resonance, imaginary part = 0

$$\Rightarrow \omega C - \frac{1}{\omega L} = 0$$

$$\omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$



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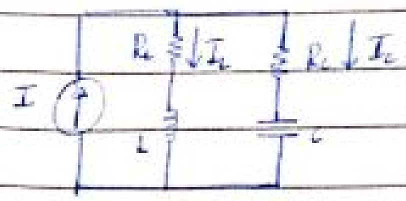
1 ii) GENERAL CASE $R_L - R_C$ parallel CIRCUIT :



The total admittance is:

$$Y = Y_{RL} + Y_{RC} \longrightarrow ①$$

$\rightarrow$ $$Y_{RL} = \frac{1}{R_L + j\omega L} \times \frac{(R_L - j\omega L)}{(R_L - j\omega L)} = \frac{R - j\omega L}{R_L^2 - (j\omega L)^2}$$

$$Y_{RL} = \frac{R - j\omega L}{R_L^2 + \omega^2 L^2}$$

$$\boxed{Y_{RL} = \frac{R}{R_L^2 + \omega^2 L^2} - j\frac{\omega L}{R_L^2 + \omega^2 L^2}} \longrightarrow ②$$

$\rightarrow$ $$Y_{RC} = \frac{1}{R_C + \frac{1}{j\omega C}} = \frac{1}{R_C - \frac{j}{\omega C}} = \frac{\omega C}{R_C\omega C - j}$$

$$Y_{RC} = \frac{\omega C}{R_C\omega C - j} \times \frac{(R_C\omega C + j)}{(R_C\omega C + j)}$$

$$= \frac{R_C\omega^2 C^2 + j\omega C}{(R_C\omega C)^2 - 1}$$

$$Y_{RC} = \frac{1}{R_C - \frac{j}{\omega C}} \cdot \frac{(R + \frac{j}{\omega C})}{(R + \frac{j}{\omega C})}$$

$$\boxed{Y_{RC} = \frac{R_C\omega^2 C^2}{(R_C\omega C)^2 - 1} + j\frac{\omega C}{(R_C\omega C)^2 - 1}} \longrightarrow ③$$

Using ② & ③ in ① ; equating imaginary part to zero, we have,

$$\frac{L\omega}{R_L^2 + \omega^2 L^2} - \frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} = 0$$

$$\frac{L\omega}{R_L^2 + \omega^2 L^2} - \frac{\omega C}{R_C^2\omega^2 C^2 + 1} = 0$$



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$$\omega L R_L^2 \omega^2 C^2 + \omega L - [\omega C R_L^2 + \omega^3 C L^2] = 0$$

$$\omega^3 R_L^2 L C^2 + \omega L - \omega C R_L^2 - \omega^3 C L^2 = 0$$

$$\omega[\omega^2 R_L^2 L C^2 + L - C R_L^2 - \omega^2 C L^2] = 0$$

$$\omega^2[R_L^2 L C^2 - C L^2] = R_L^2 C - L$$

$$\boxed{\omega^2 = \frac{R_L^2 C - L}{[R_L^2 L C^2 - C L^2]}}$$

$$\omega = \sqrt{\frac{R_L^2 C - L}{R_L^2 L C^2 - C L^2}}$$

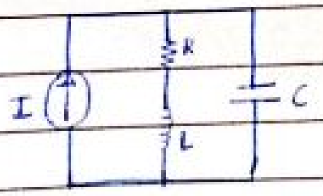
$$\omega = \sqrt{\frac{R_L^2 C - L}{LC(R_C^2 C - L)}} = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}}$$

$$\boxed{\omega = \omega_0 \sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}}$$

$$\Rightarrow \boxed{f = f_0 \sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}} \quad \text{or} \quad \boxed{f_p = f_s \sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}}$$

found in 1st derivation. parallel series

## 3) Practical C - RL Parallel Circuit:




The total admittance is:

$$Y = Y_{RL} + Y_C$$

$$Y = \frac{1}{R_L + j\omega L} + \frac{1}{1/j\omega C}$$

$$Y = \frac{R}{R_L^2 + \omega^2 L^2} - \frac{j\omega L}{R_L^2 + \omega^2 L^2} + j\omega C$$

from previous derivation.



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Equating imaginary part to zero.

$$\omega C - \frac{\omega L}{R^2 + \omega^2 L^2} = 0$$

$$\omega C = \frac{\omega L}{R^2 + \omega^2 L^2}$$

$$R^2 C + \omega^2 L^2 C = L$$

$$\omega^2 L^2 C = L - R^2 C$$

$$\omega^2 = \frac{L - R^2 C}{L^2 C}$$

$$\omega^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$\omega = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{L}}$$

$$\omega = \omega_0 \sqrt{1 - \frac{R^2 C}{L}}$$

$$\boxed{f = f_0 \sqrt{1 - \frac{R^2 C}{L}}}$$

(or)

$$\boxed{f_p = f_0 \sqrt{1 - \frac{R^2 C}{L}}}$$



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- IMPEDENCE OF ANTI-RESONANT CIRCUIT [DYNAMIC RESISTANCE]

For Case 3),

$$Y = \frac{R}{R^2+L^2\omega^2} + j\left[\omega C - \frac{\omega L}{R^2+L^2\omega^2}\right]$$

At Resonance, imaginary part is zero.

$$Y_0 = \frac{R}{R^2+L^2\omega_0^2}$$

$$Y_0 = \frac{R}{R^2+\frac{L^2}{L^2C^2}} = \frac{R}{R^2+\frac{L}{C}}$$

If $R^2 << \frac{L}{C}$, then,

$$Y_0 = \frac{R}{L/C} = \frac{RC}{L}$$

$$\boxed{Z_0 = \frac{L}{RC}}$$

—x—END—x—



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13-11-19

# MODULE - 4

# LAPLACE TRANSFORMS

DEFINITION:

$$F(s) = L\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt.$$

- INVERSE LAPLACE:

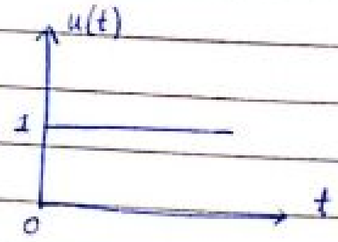
$$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)\, e^{st}\, ds$$

- SINGULARITY FUNCTIONS:

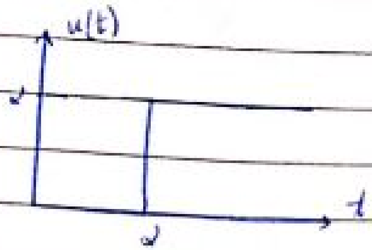
1) Unit-step Function
2) Ramp
3) Delta

1) UNIT STEP FUNCTION:

→ $u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$




→ $2u(t-2)$



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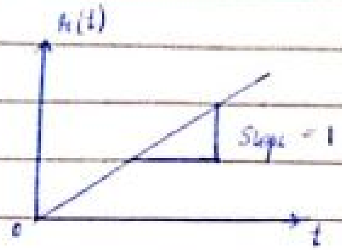
②



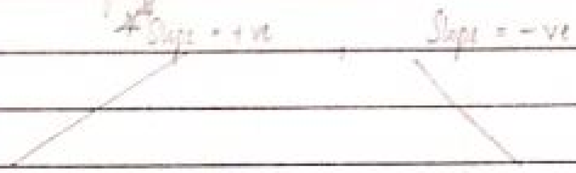
2) Ramp Function:




→ $r(t) = \begin{cases} t & , t \geq 0 \\ 0 & , t < 0 \end{cases}$

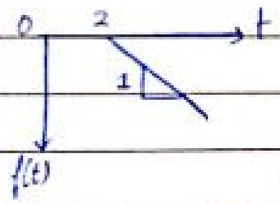
$$\text{Slope} = \tan\theta = \frac{Y}{X} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

→ $r(t) = \text{slope} \times (t)$ (= 1)
$= \text{Slope} \times (t) \cdot u(t)$ ($\because u(t) = 1$ for $t > 0$)




→ $r(t-1) = \text{slope} \times (t-1) \cdot u(t-1)$

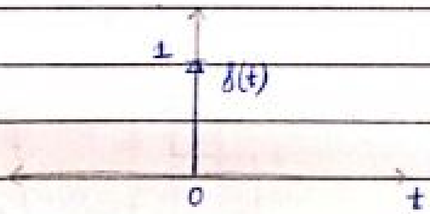
Ex:



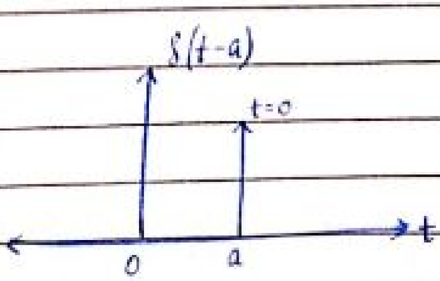

$f(t) = -1\, r(t-2)$

3) DELTA FUNCTION:

→ $\delta(t) = \begin{cases} 1 & , t = 0 \\ 0 & , t \neq 0 \end{cases}$




→ $\delta(t-a)$

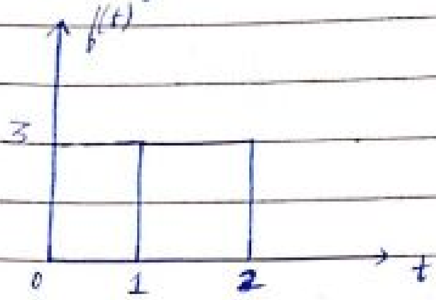


③



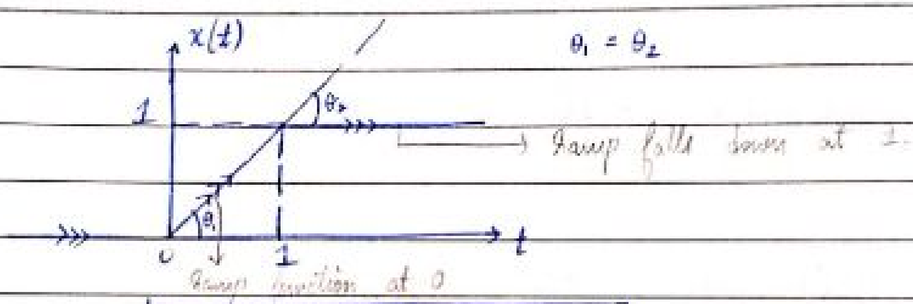
• Express the given waveforms in terms of $u(t)$ and $r(t)$

1)




$$\boxed{f(t) = \underbrace{3\,u(t-1)}_{\text{rise}} - \underbrace{3\,u(t-2)}_{\text{fall}}}$$

2)

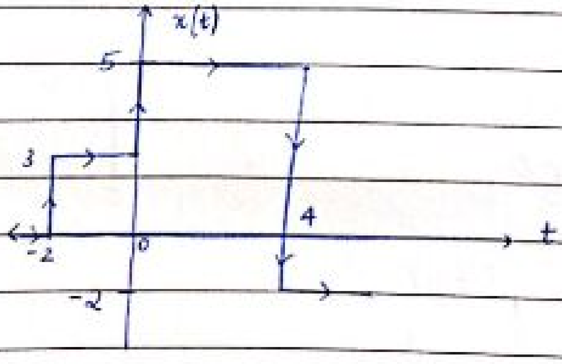



$$\boxed{x(t) = 1\,r(t) - 1\,r(t-1)}$$

$$x(t) = 1.t.u(t) - 1.(t-1)\,u(t-1)$$

$[\because r(t) = \text{slope} \times t \times u(t)]$

3)




$$x(t) = 3u(t+2) + 2u(t) - 5u(t-4) - 2u(t-4)$$

$$\Rightarrow \boxed{x(t) = 3u(t+2) + 2u(t) - 7u(t-4)}$$



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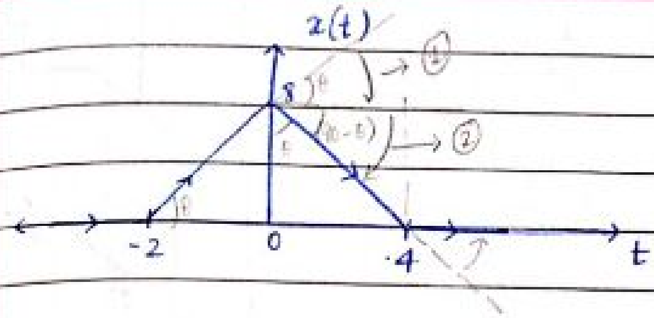
④

4)




(0,8), (-2,0) ⇒ $\frac{8-0}{0+2} = 4$

(0,8) (4,0) ⇒ $\frac{8-0}{0-4} = -2$

$x(t) = 4\,r(t+2) - 2\,r(t-4)$

Starting point of ①

$x(t) = 4\,r(t+2) - 4\,r(t) - 2\,r(t) + 2\,r(t-4)$

↳ Starting point of ②.

$\Rightarrow \boxed{x(t) = 4\,r(t+2) - 6\,r(t) + 2\,r(t-4)}$

(B)

$\boxed{x(t) = 4\,(t+2)u(t+2) - 6\,t \times u(t) + 2\,(t-4)\,u(t-4)}$

→ Always just consider starting point.

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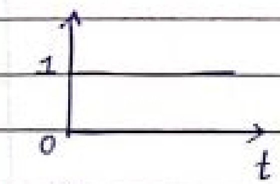
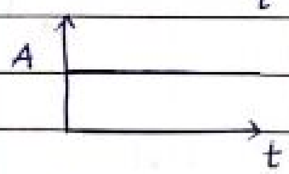
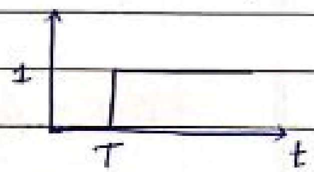
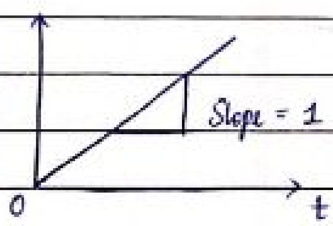
| Function, $f(t)$ | Laplace Transform, $F(s)$. | WAVEFORM |
|---|---|---|
| 1) $u(t)$ [Unit Step function] | $\frac{1}{S}$ |  |
| 2) $A\,u(t)$ | $\frac{A}{S}$ |  |
| 3) Delayed unit step $u(t-T)$ | $\frac{e^{-TS}}{S}$ |  |
| 4) Ramp input, $r(t) = t\,u(t)$ | $\frac{1}{S^2}$ |  |



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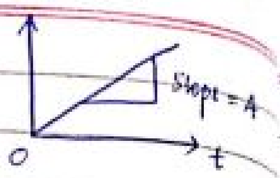
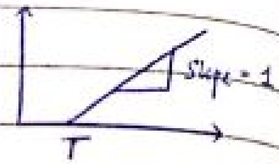
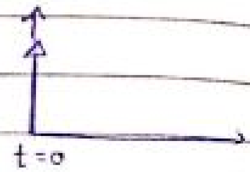
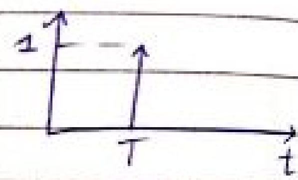
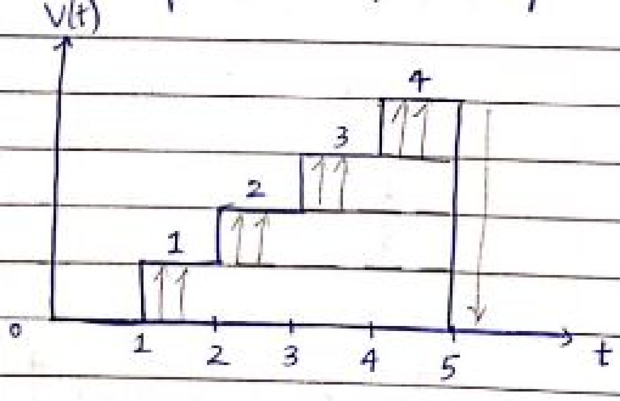
| | | | |
|---|---|---|---|
| 5) | $A\,t\,u(t)$ | $\frac{A}{s^2}$ |  |
| 6) | Delayed unit ramp. $r(t-T)=(t-T)\,u(t-T)$ | $\frac{e^{-Ts}}{s^2}$ |  |
| 7) | Delta, $\delta(t)$ [Unit impulse] | $1$ |  |
| 8) | Delayed unit impulse $\delta(t-T)$ | $e^{-Ts}$ |  |

1) For the given waveform, obtain the laplace transform.




$$V(t) = u(t-1) + u(t-2) + u(t-3) + u(t-4) - 4\,u(t-5)$$

$$\boxed{V(t) = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + \frac{e^{-4s}}{s} - \frac{4e^{-5s}}{s}}$$



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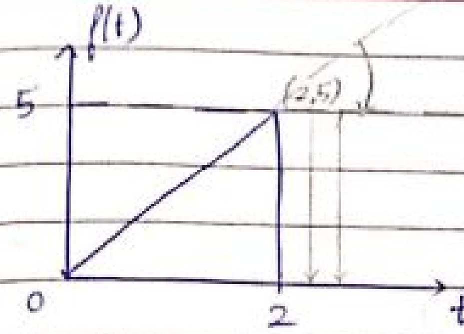
6

2) 




Slope $= \frac{5}{2}$

S:

$$f(t) = \frac{5}{2} r(t) - \frac{5}{2} r(t-2) - 5u(t-2)$$

$$F(s) = \frac{5}{2}\frac{1}{s^2} - \frac{5}{2}\frac{e^{-2s}}{s^2} - \frac{5e^{-2s}}{s}$$

$$\boxed{F(s) = \frac{5(1-e^{-2s})}{2s^2} - \frac{5e^{-2s}}{s}}$$

(d)

3) $\boxed{f(t) = x(t) \times g(t)}$ ⟶ gating pulse.

$g(t) = u(t) - u(t-2)$ [The window from $0 \longrightarrow 2$]

$y = x(t) = mx + c = \frac{5}{2}\cancel{t} + 0$

$$\boxed{x(t) = \frac{5}{2}t}$$

(1) ⇒ $f(t) = \frac{5}{2}t \times [u(t) - u(t-2)]$

$= \frac{5}{2}t\,u(t) - \frac{5}{2}t\,u(t-2)$

$= \frac{5}{2}t\,u(t) - \frac{5}{2}(t-2+2)\,u(t-2)$ [Add & subtract 2 to 't']

$$\boxed{f(t) = \frac{5}{2}t\,u(t) - \frac{5}{2}(t-2)u(t-2) - 5u(t-2)}$$



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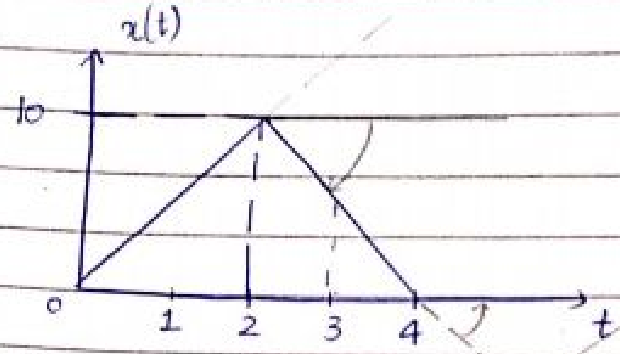
3)




(2,10), (4,0)

$$\text{Slope} = \frac{10-0}{4-2} = \frac{10}{2}$$

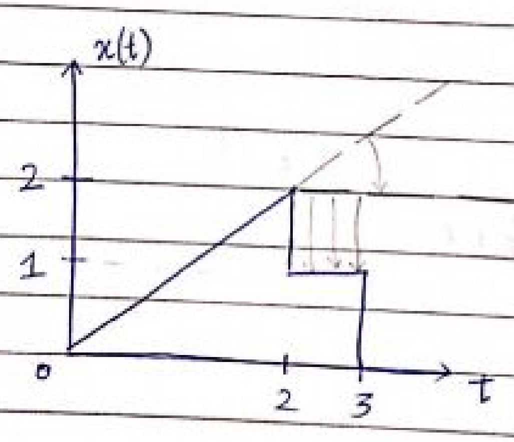
$$x(t) = \frac{10}{2} r(t) - \frac{10}{2} r(t-2) - \frac{10}{2} r(t-2) + \frac{10}{2} r(t-4)$$

$$x(t) = 5r(t) - 10r(t-2) + 5r(t-4)$$

$$x(t) = \frac{5}{s^2} - \frac{10e^{-2s}}{s^2} + \frac{5e^{-4s}}{s^2}$$

$$\boxed{x(t) = \frac{5 - 10e^{-2s} + 5e^{-4s}}{s^2}}$$

* 4)




S:

$$x(t) = \frac{2}{2} r(t) - \frac{2}{2} r(t-2) - 1u(t-2) - 1u(t-3)$$

$$x(t) = r(t) - r(t-2) - u(t-2) - u(t-3)$$

$$\boxed{x(t) = \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}}$$



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5) • Laplace transform of PERIODIC FUNCTION

Consider a periodic function of time period, T satisfying the condition,

$$\boxed{f(t+nT) = f(t)}$$

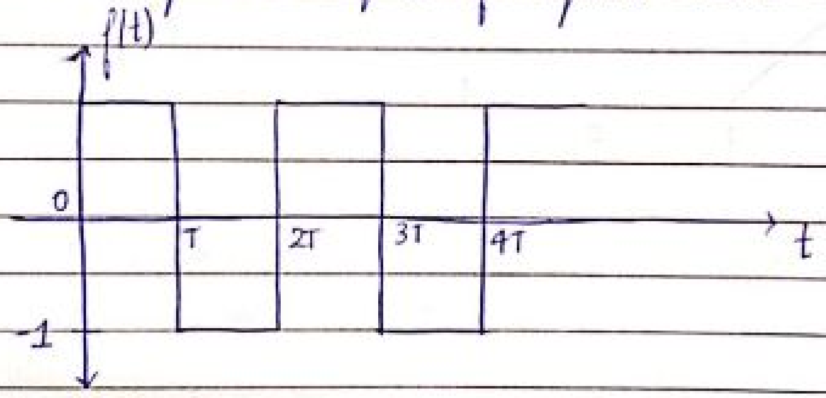
where, 'n' is a positive or negative integer.

→ Laplace transform of such a periodic function is given by

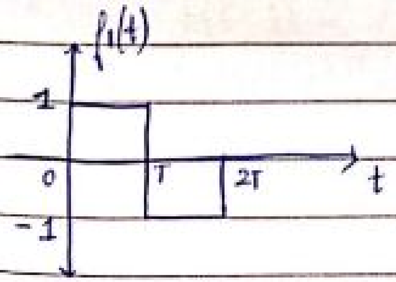
$$\boxed{F(s) = \frac{F_1(s)}{1-e^{-sT}}}$$

→ $F_1(s)$ is the Laplace transform of first cycle of the periodic function.

1) Obtain the Laplace transform of square wave train




S:




$$\boxed{f_1(t) = u(t) - 2u(t-T) + u(t-2T)}$$

$$\boxed{F_1(t) = \frac{1}{s} - \frac{2e^{-Ts}}{s} + \frac{e^{-2Ts}}{s}}$$

From the formula, $F(s) = \dfrac{F_1(s)}{1-e^{-sT}}$

Here, $T = 2T$



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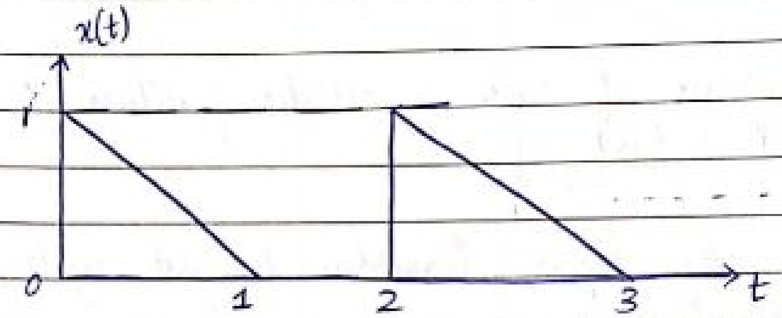
(9)

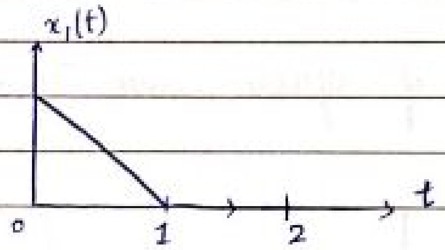
$$F(s) = \frac{\frac{1}{s} - \frac{2e^{-Ts}}{s} + \frac{e^{-2Ts}}{s}}{1 - e^{-2sT}}$$

$$\boxed{F(s) = \frac{1 - 2e^{-Ts} + e^{-2Ts}}{s\left(1 - e^{-2sT}\right)}}$$

2)




S:




$x_1(t) = y(t) \times g(t) \longrightarrow (1)$

$y(t) = mx + c$

$= -\frac{1}{1} x + 1$

$g(t) = u(t) - u(t-1)$

its '1' as its dead after 1

$\boxed{y(t) = -(t-1)}$

$x_1(t) = -(t-1)\left[u(t) - u(t-1)\right]$

$x_1(t) = + u(t) - t\,u(t) + (t-1)\,u(t-1)$

$x_1(t) = u(t) - t\,u(t) + (t-1)\,u(t-1)$

$$\boxed{X_1(s) = \frac{1}{s} - \frac{1}{s^2} + \frac{e^{-s}}{s^2}}$$



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$$F(s) = \frac{x_1(s)}{1-e^{-sT}}$$

$$F(s) = \frac{\frac{1}{s} - \frac{1}{s^2} + \frac{e^{-s}}{s^2}}{1-e^{-2s}}$$

$$\boxed{F(s) = \frac{s(1+e^{-s}) - 1}{s^2(1-e^{-2s})}}$$

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- **INITIAL VALUE THEOREM:**

$$f(0^+) = \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} [sF(s)]$$

The only restriction is that $f(t)$ must be continuous or at the most a step discontinuity at $t=0$.

PROOF:

We know that, Laplace transform of:

$$L[f'(t)] = sF(s) - f(0^-)$$

Using the standard form of laplace Transform:

$$\int_0^{\infty} f'(t)\, e^{-st}\, dt = sF(s) - f(0^-)$$

[Applying $\lim_{s \to \infty}$ on both sides]

$$\underbrace{\lim_{s \to \infty} \int_0^{\infty} f'(t)\, e^{-st}\, dt}_{} = \lim_{s \to \infty} [sF(s) - f(0^-)]$$

$$0 = \lim_{s \to \infty} [sF(s) - f(0^-)]$$



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$$\Rightarrow \lim_{s\to\infty} [sF(s)] = f(0^-)$$

$$f(0^-) = f(0^+).$$

Thus, $\boxed{f(0^+) = \lim_{s\to\infty} [sF(s)]} = f(0^-)$

or

$$\boxed{\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} [sF(s)]}$$

- FINAL VALUE THEOREM: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} s[F(s)]$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} [sF(s)]$$

PROOF: We know that,

$$L[f'(t)] = sF(s) - f(0^-)$$

$$\int_0^\infty f'(t) e^{-st}\, dt = sF(s) - f(0^-)$$

Applying $\lim_{s\to 0}$ on both sides.

$$\lim_{s\to 0} \int_0^\infty f'(t)\, e^{-st}\, dt = \lim_{s\to 0} sF(s) - f(0^-)$$

$$\Rightarrow \int_0^\infty f'(t)\, dt = \lim_{s\to 0} sF(s) - f(0^-)$$

$$f(t)\Big]_0^\infty = \lim_{s\to 0} sF(s) - f(0^-)$$



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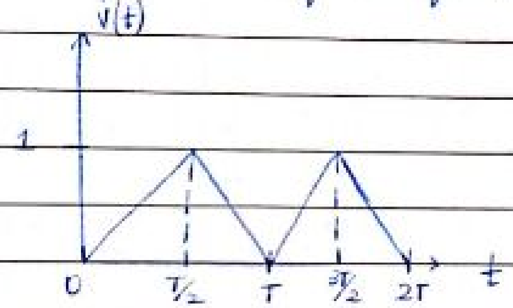
$$f(\infty) - f(0) = \lim_{s \to 0} sF(s) \cdot f(0^-)$$

$$\boxed{f(\infty) = \lim_{s \to 0} sF(s)}$$

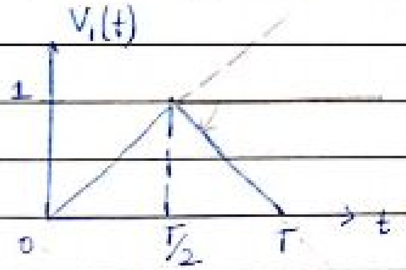
(or)

$$\boxed{\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)}$$

1) Find the Laplace Transform of the periodic function




S:




$$V_1(t) = \frac{2}{T} r(t) - \frac{2}{T} r\left(t - \frac{T}{2}\right) - \frac{2}{T} r\left(t - \frac{T}{2}\right) + \frac{2}{T} r(t - T)$$

$$V_1(t) = \frac{2}{T} r(t) - \frac{4}{T} r\left(t - \frac{T}{2}\right) + \frac{2}{T} r(t - T)$$

$$V_1(s) = \frac{2}{T} \times \frac{1}{s^2} - \frac{4}{T} \frac{e^{-\frac{T}{2}s}}{s^2} + \frac{2}{T} \frac{e^{-Ts}}{s^2}$$

$$\boxed{V_1(s) = \frac{2}{s^2 T} \left[1 - 2e^{-\frac{Ts}{2}} + e^{-Ts}\right]}$$



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$$F(s) = \frac{V_1(s)}{1-e^{-sT}} \qquad [T = T]$$

$$\boxed{F(s) = \frac{2\left[1-2e^{-Ts/2}+e^{-Ts}\right]}{Ts^2\left(1-e^{-Ts}\right)}}$$

2) Find $f(0)$ and $f(\infty)$ for the given Transfer function.

$$F(s) = \frac{s^3+7s^2+5}{s(s^3+3s^2+4s+2)}$$

S:

Using Initial value theorem,

$$f(0) = \lim_{s\to\infty} s\,F(s)$$

$$f(0) = \lim_{s\to\infty} s\left[\frac{s^3+7s^2+5}{s(s^3+3s^2+4s+2)}\right]$$

$$f(0) = \lim_{s\to\infty} \frac{s^3\left[1+\cancel{\frac{7}{s}}^{0}+\cancel{\frac{5}{s^3}}^{0}\right]}{s^3\left[1+\cancel{\frac{3}{s}}^{0}+\cancel{\frac{4}{s^2}}^{0}+\cancel{\frac{2}{s^3}}^{0}\right]} = \frac{1}{1}$$

$$\Rightarrow \boxed{f(0) = 1}$$

$$f(\infty) = \lim_{s\to 0} s\,F(s) \qquad \text{[Using Final Value theorem]}$$

$$= \lim_{s\to 0} s\left[\frac{s^3+7s^2+5}{s(s^3+3s^2+4s+2)}\right]$$

$$= \frac{0+0+5}{0+0+0+2}$$

$$\boxed{f(\infty) = \frac{5}{2}}$$



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3) $F(s) = \frac{s(s+4)(s+8)}{(s+1)(s+6)}$. Find $f(0)$ & $f(\infty)$.

S: Using initial value problem,

$$f(0) = \lim_{s\to\infty} sF(s)$$

$$= \lim_{s\to\infty} s\left[\frac{s(s+4)(s+8)}{(s+1)(s+6)}\right]$$

$$= \lim_{s\to\infty} s^2\left[\frac{s^2+12s+32}{s^2+7s+6}\right]$$

$$= \lim_{s\to\infty} \frac{s^2 s^2}{s^2}\left[\frac{1+12/s+32/s^2}{1+7/s+6/s^2}\right]$$

$$= \infty\left[\frac{1+0+0}{1+0+0}\right]$$

$$\boxed{f(0) = \infty}$$

$$f(\infty) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} s\left[\frac{s(s+4)(s+8)}{(s+1)(s+6)}\right]$$

$$\boxed{f(\infty) = 0}$$

4) $F(s) = \frac{(s+2)(e^{2s})}{s^2+5}$

S: $f(0) = \lim_{s\to\infty} s\left[\frac{(s+2)e^{+2s}}{s^2+5}\right]$

$$= \lim_{s\to\infty} \frac{s^2}{s^2}\left[\frac{(1+2/s)e^{+2s}}{1+5/s^2}\right]$$

$$= \frac{(1+0)(e^{+2(\infty)})}{1+0}$$

$$\boxed{f(0) = \infty}$$

$$f(\infty) = \lim_{s\to 0} s\left[\frac{(s+2)e^{2s}}{s^2+5}\right]$$

$$= 0\left[\frac{2e^0}{5}\right]$$

$$\boxed{f(\infty) = 0}$$



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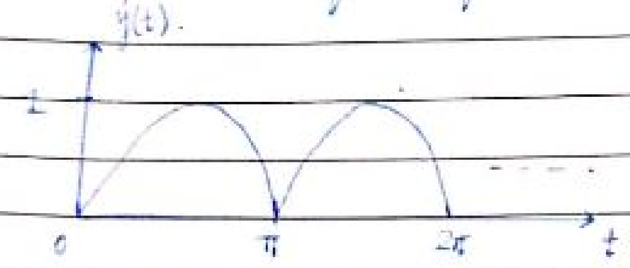
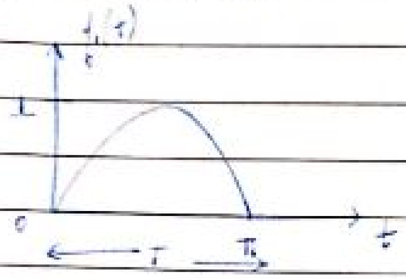
5) Find the Laplace transform of



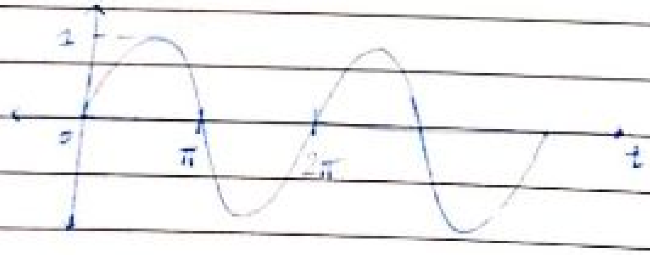

S: Consider the first cycle.

~~y(t)~~ → $\boxed{T=\pi}$.




→ $f_1(t)$ is made up of two sinusoidal waveforms.

→ Consider ~~in~~ a sinusoidal waveform with time period $T = 2\pi$.




$$f_a(t) = \sin(\omega t)\, u(t)$$

→ $\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{2\pi} \Rightarrow \boxed{\omega = 1}$ $[\because T = 2\pi]$

$\Rightarrow \boxed{f_a(t) = \sin t\, u(t)}$

Now, it is necessary to cancel the cycles [half cycles] after $\boxed{T=\pi}$



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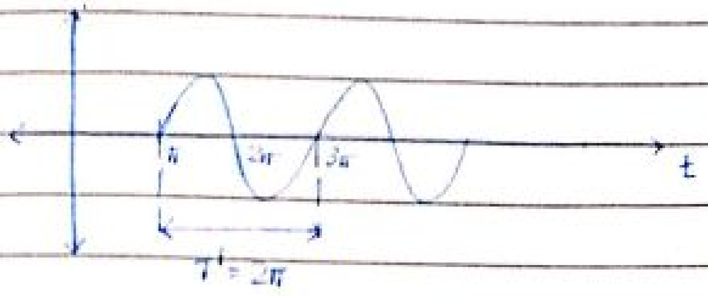
So, consider the shifted sine wave with period $T' = 2\pi$.




$$f_b(t) = \sin(t-\pi)\, u(t-\pi)$$

$$\therefore\ f_1(t) = f_a(t) + f_b(t)$$
$$= \sin t\, u(t) + \sin(t-\pi)\, u(t-\pi)$$

$$\boxed{f_1(s) = \frac{1}{s^2+1}(1) + \frac{e^{-\pi s}}{s^2+1} \times (1)}$$

$$F(s) = \frac{f_1(s)}{1 - e^{-sT}}$$

$$\boxed{F(s) = \frac{(1 + e^{-\pi s})}{(1+s^2)(1 - e^{-\pi s})}}$$

$$\boxed{F(s) = \frac{\cosh}{(s^2+1)}\left(\frac{\pi s}{2}\right)}$$



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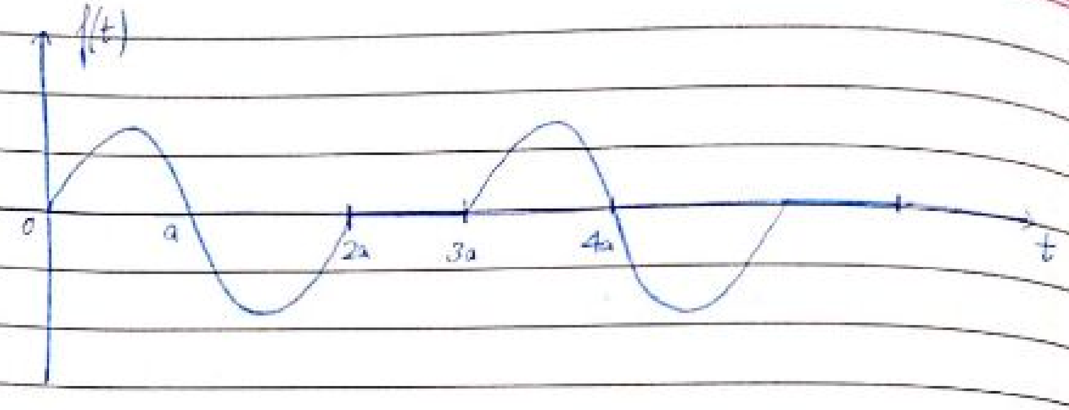
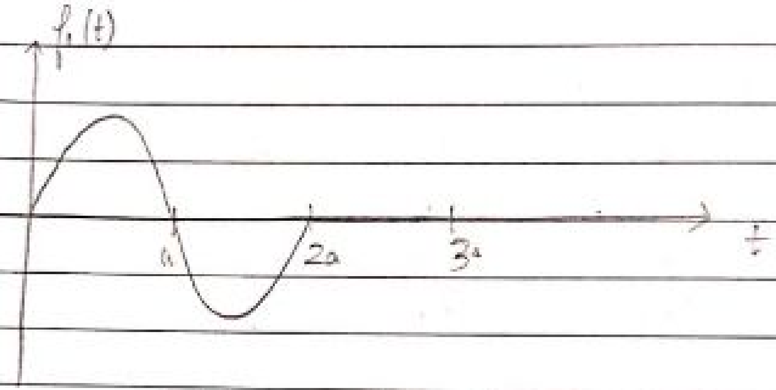
(17)

28-11-19 1)




S.




$f_a(t) = \sin(\omega t)\, u(t)$ $\quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2a}$

$$\boxed{f_a(t) = \sin\left(\frac{\pi t}{a}\right) u(t)}$$

To get a dead line (2a)

$f_b(t) = -\sin \omega(t-2a)\, u(t-2a)$ $\quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2a}$

$f_b(t) = -\sin \frac{\pi(t-2a)}{a}\, u(t-2a)$

$$f_1(t) = f_a(t) + f_b(t)$$

$$f_1(t) = \sin\left(\frac{\pi t}{a}\right) u(t) - \sin \frac{\pi}{a}(t-2a)\, u(t-2a)$$

$$f_1(s) = \frac{(\pi/a)}{s^2 + (\frac{\pi}{a})^2} - \frac{e^{-2as}}{s^2 + (\pi/a)^2}$$

$$F_s = \frac{f_1(s)}{1 - e^{-Ts}} \quad (T = 3a).$$



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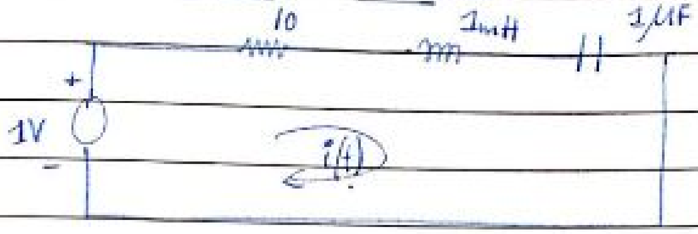
$$\boxed{F(s) = \frac{f_1(s)}{1 - e^{-3as}}}$$

Q) Using laplace transform, obtain an expression for current. Assume zero initial conditions.




S: Applying KVL to the given loop:

$$1 - 10i(t) - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)dt = 0$$

$$\Rightarrow \quad 1 = 10i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

Applying laplace:

$$\frac{1}{s} = 10I(s) + L\left[sI(s) - \cancel{I(0^-)}^{\,0}\right] + \frac{1}{C}\left[\frac{1}{s}I(s)\right]$$

$$\frac{1}{s} = 10I(s) + LsI(s) + \frac{1}{Cs}I(s)$$

$$\frac{1}{s} = I(s)\left[10 + Ls + \frac{1}{Cs}\right]$$

$$\frac{1}{\cancel{s}} = I(s)\left[\frac{10Cs + LCs^2 + 1}{C\cancel{s}}\right]$$

$$\boxed{I(s) = \frac{C}{10Cs + LCs^2 + 1}}$$

—x— END —x—



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